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GERARD OF CREMONA'S TRANSLATION OF AL-KHWĀRIZMĪ'S *AL-JABR*: A CRITICAL EDITION

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THE most significant mathematical innovations of the high Middle Ages were the introduction of algebra into Western Europe through the translations of al-Khwārizmī's *al-Kitāb al-mukhataṣar fī ḥisāb al-jabr wa'l-muqābala* (*Liber algebre et almuchabala*) and the foundation of abacist arithmetic in the *Liber abaci* by Leonardo da Pisa.[1] The latter work has been subjected to considerable study;[2] more is certainly warranted. The translations of *Ḥisāb al-jabr*, however, have long been in a process of sorting and study. In 1838 Guillaume Libri published a faulty edition of Gerard's translation.[3] Twelve years later Prince Baldassarre Boncompagni presented a transcription of William of Lunis' translation which the Prince incorrectly accepted as that of Gerard of Cremona.[4] Louis Karpinski in 1915 offered a critical edition of a late copy of Robert of Chester's translation.[5] Several years ago I reported on sixteen copies of the three translations;[6] and in the near future my new critical edition

[1] B. Boncompagni, *Scritti di Leonardo Pisano...* 1 (Rome, 1857), pp. 1-459.

[2] Apart from studies in standard histories of mathematics, e.g., M. Cantor, *Vorlesungen über Geschichte der Mathematik* 1 (Leipzig, 1894), pp. 676-89, useful information may be found in K. Vogel, 'Fibonacci, Leonardo', *Dictionary of Scientific Biography* 4 (New York, 1971), pp. 604-13 (hereafter cited as *DSB*), particularly for the bibliography, and B. Boncompagni, 'Della vita e delle opere di Leonardo Pisano matematico del secolo decimoterzo', *Atti dell' Accademia pontificia de'nuovi lincei* 5.1-3 (1851-52) (hereafter cited as *Atti*), which describes the codices containing *Liber abaci*.

[3] G. Libri, *Histoire des sciences mathématiques en Italie depuis la renaissance des lettres jusqu'à la fin du xvii^e^ siècle* 1 (Paris, 1838), pp. 253-97.

[4] B. Boncompagni, 'Della vita e delle opere di Gherardo Cremonese, traduttore del secolo duodecimo...', *Atti* 4 (1850-51) 412-35.

[5] L. Karpinski, *Robert of Chester's Latin Translation of the Algebra of Al-Khowarizmi* (New York, 1915). Note here a second spelling for 'al-Khwārizmī' and there are more; see for instance n. 8 below. My preference is based on the spelling used in *DSB* 7.358.

[6] B. Hughes, 'The Medieval Latin Translations of al-Khwārizmī's *al-jabr*', *Manuscripta* 26 (1982) 31-37. This article corrects and adds to F. J. Carmody's *Arabic Astronomical and Astrological Sciences in Latin Translation. A Critical Bibliography* (Berkeley, 1956), pp. 47-48.

of Robert of Chester's translation will be published.[7] The core of the present article is a critical edition of the oldest extant copy of the translation made by Gerard, together with variants found in the three older manuscripts which reproduce it most faithfully. Remarks about the other manuscript copies and an analysis of the tract complete the article.

Analysis of the Text

According to the translation the treatise is divided into eight chapters and an appendix. These discuss in turn decimal and algebraic numbers, six canonical first and second degree equations, geometric demonstrations for three quadratic solutions, methods for multiplying with binomials, computing with roots, further examples for each type of equation, a variety of algebraic problems, business problems involving proportion, and (as an appendix) additional problems illustrating some of the standard equations. In the following analysis of each section the discussion will employ modern terminology, such as constant and coefficient, rather than the labored phraseology of the translator. The text of the latter can always be consulted to appreciate the efforts made and success realized by al-Khwārizmī as he sought to put new concepts and techniques in old words.

One may well wonder if al-Khwārizmī had forgotten that he had written a tract on the decimal system entitled *On Hindu Numerals*.[8] There he acknowledged that the decimal system originated with the Hindus; but here in the *Liber algebre* he credits himself with the discovery. As for algebraic numbers al-Khwārizmī set the terminology: square (*census*), root (*radix*) and constant (two names: *numerus simplex* and *dragma*). While he may have developed these ideas from a study of Diophantos' *Arithmetic* or Euclid's *Elements*,[9] no one may

[7] *Robert of Chester's Latin Translation of al-Khwārizmī's AL-JABR. A New Critical Edition* (forthcoming). Unlike Karpinski's edition, mine is based on the oldest Latin manuscripts.

[8] Edited by B. Boncompagni, *Trattati d'arithmetica*, vol. 1: *Algoritmi de numero Indorum* (Rome, 1857), and by K. Vogel, *Mohammed ibn Musa Alchwarizmi's Algorismus. Das früheste Lehrbuch zum Rechnen mit indischen Ziffern* (Aalen, 1963).

[9] T. L. Heath, *Diophantos of Alexandria. A Study in the History of Greek Algebra*, 2nd edition (Cambridge, 1910) and (trans.) *The Thirteen Books of Euclid's Elements*, 2nd rev. edition, 3 vols. (Cambridge, 1926; rpt. New York, 1956); and J. L. Heiberg and E. S. Stamatis, eds., *Euclidis Elementa*, 2nd rev. edition, 5 vols. (Leipzig, 1969-73). The question of al-Khwārizmī's sources was addressed by Gandz who describes three schools of thought: Hindu influence, Greek (or Greek-Hindu) resources, and Syriac-Persian fonts. He explicitly rules out the Greek background because he claims that Diophantos' *Arithmetica*, the most likely source of theory and problems for al-Khwārizmī, was translated only after the latter's death. Gandz prefers the Syriac-Persian fonts. Hartner, on the other hand, who presents an informative description of the development of algebra in Islamic lands, opts for the Greek resource, particularly that of Diophantos. His opinion is in line with that of Rodet who claims that al-Khwārizmī was 'purely and simply a disciple of the Greek school'. See S. Gandz, 'The Sources of al-Khowarizmi's Algebra', *Osiris* 1 (1936) 263-77; W. Hartner, 'Djabr', *The Encyclopaedia of Islam*, new edition, 5 (Leiden, 1965), pp. 360-62; L. Rodet, 'L'Algèbre d'Al-Khârizmi et les méthodes indienne et grecque', *Journal asiatique* 11 (1878) 5-98.

deny him the creation of an entirely new approach to problem solving, the standardization of types of equations.

Combining variously three algebraic numbers, al-Khwārizmī constructs six types of equations, three which we will call *simple*, since he himself labeled the last three *composite*. They are:

simple:	$ax^2 = bx$
	$ax^2 = c$
	$bx = c$
composite:	$ax^2 + bx = c$
	$ax^2 + c = bx$
	$bx + c = ax^2$.

The three simple equations are exemplified and solved with dispatch:

$x^2 = 5x$	$x = 5$
$5x^2 = 80$	$x^2 = 16$
$\frac{1}{2}x = 10$	$x = 20$.

Apparently the student was expected to memorize the paradigms, for no explicit rules are offered for solving simple equations, save one: if the coefficient of the unknown is greater or less than unity, divide or multiply all terms by the inverse of the coefficient to reach unity.[10] A geometric structure supports all these equations, both simple and composite, as the proofs of the methods show. The strategy of setting one side of an equation equal to zero did not occur until the seventeenth century;[11] the thinking of al-Khwārizmī and his successors aligned number with geometric magnitude, a concept difficult to dispose of.[12]

The first example for composite equations is the oft-quoted $x^2 + 10x = 39$ and it is solved by completing the square. The root 3 is found, of course, but it is not the unknown; the unknown is the square, 9. In other problems the unknown is the root. Al-Khwārizmī seems to want his readers to be flexible in

[10] See below, II.A.11-12: 'Similiter quoque quod fuerit maius censu aut minus, ad unum reducetur censum.' (References are to chapter and line numbers of the text edited on pp. 233-61) below).

[11] Credit for first setting an equation equal to zero probably belongs to Thomas Harriot (1560-1621), author of *Artis analyticae praxis*... (London, 1631); see G. Loria, *Storia delle matematiche dall'alba della civiltà al tramonto del secolo xix*, 2nd edition (Milan, 1950), p. 445, and J. Wallis, *A Treatise of Algebra, Both Historical and Practical*... (London, 1685), p. 198.

[12] A. G. Molland, 'An Examination of Bradwardine's Geometry', *Archive for the History of the Exact Sciences* 19 (1978) 113-75.

what is to be sought. Following the example he reiterates the need to reduce, where necessary, the coefficient of the squared term to unity. Three additional problems exemplify the first of the composite types, all solved by completing the square. It should be noted that al-Khwārizmī had no word for *coefficient* and that he expects his readers to understand that 'Media igitur radices' means 'Halve the coefficient of the second degree term'. Moreover, he uses the word *questio* to signify our term *equation*.

Within the explanation accompanying the solution of the second type of composite equation, al-Khwārizmī discusses whether or not an equation in the form $ax^2 + c = bx$ can be solved. He says that if the square of half the coefficient of the first degree term is less than the constant, the solution is impossible. Furthermore, he remarks, if the same square equals the constant, then the root is immediately equal to half the coefficient.[13] All of this, obviously, is a beginning of an analysis of the discriminant, $\sqrt{b^2 - 4ac}$. Additionally, and for the first time, he observes that there may be a second root to an equation, which the student may find if he wishes: 'Quod si volueris....'

Mindful of the foregoing remarks, al-Khwārizmī shows that he is a careful teacher as he explains how to solve each of the three types of equations. The rules are easily followed and well exemplified; in fact, a certain commonality among the steps becomes obvious. Regardless of the type, the first two steps are the same: halve the number of roots and square the half. Then, for the first and third types exemplified by $x^2 + 10x = 39$ and $x^2 = 3x + 4$ respectively, the constant term is added to the square; for the second type such as $x^2 + 21 = 10x$, the constant term is subtracted from the square. Hence, as noted above, if the subtraction cannot be done, the equation cannot be solved. (Only much later, in the sixteenth century, would Cardano begin to tinker with what Descartes would call imaginary numbers whereby the second type can always have a solution.) The fourth step is the same for all types: take the square root of the sum or difference. Only the fifth and last step which directly produces the value of x is unique for each type. For the first type, subtract the half of the number of roots from the fourth step; for the second, subtract the square root from the half; and for the third, add the half to the root. Clear, complete and concise: the rules need only be memorized. In the next section the student comes to realize why the process always produces a solution. The didactic technique employed by al-Khwārizmī, therefore, is first to familiarize students with the canonical types of equations and methods for solving them and then, after some expertise had been realized, to demonstrate the reliability of the methods.

[13] See below, II B.53-56.

The meaning he intends for the word *demonstrate*, which appears as 'quod demonstrare voluimus' at the end of the last two proofs, is made explicit by the word which introduces the unit, *causa*. Rather than offer Euclidean proofs for the methods, al-Khwārizmī constructs a framework that shows visibly why the methods produce the results. His approach is in fact pedagogical (to bring understanding) rather than logical (to order understanding). All of this becomes obvious in an analysis of one demonstration.

Underlying the demonstration for the method of solving equations of the second composite type is book 2, proposition 5 of Euclid's *Elements*: 'If a straight line be cut into equal and unequal segments, the rectangle contained by the unequal segments of the whole together with the square on the straight line between the parts of the section is equal to the square on the half.'[14] Euclid of course proves the theorem synthetically; al-Khwārizmī on the contrary reaches it analytically. The proposition is illustrated in the text edited below (III, p. 239); but it must be observed that the diagram is a composite picture showing all the steps together. The reader is expected to draw the figure step by step in order to appreciate the force of the demonstration. Here is how one should proceed to solve $x^2 + 21 = 10x$:

(1) Construct a square to represent the area of x^2; (2) attach a rectangle to a side of the square to represent the area 21; (3) thus added together by juxtaposition, the two areas equal the area of a rectangle of dimensions 10 by x, as shown in fig. 1.

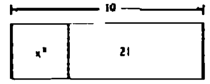


Fig. 1

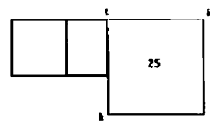


Fig. 2

(4) Bisect the side of length 10 at *t* and on the half construct square *tklg* (fig. 2) whose area is 25. (5) On *hk* (fig. 3) construct the square *hkmn*.

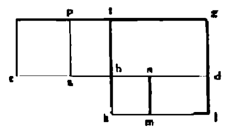


Fig. 3

With the constructions complete, al-Khwārizmī leads the reader through a chain of reasoning which I will abbreviate.[15] The area of rectangle *ahtp* equals the area of rectangle *mldn*, and therefore the area of composite figure *thnmlg* equals 21. Hence the area of square *kmnh* is 4, and segment $hk = ah = 2$. But since $eh = ea + ah = x + ah = 5$, then $x = 3$. If the length of segment x is known, the area of the previously unknown square x is 9. And that was what was sought; the demonstration is complete. Through a series of visible constructions and a sequence of logical steps, therefore, the student has been led

[14] Heath, *Elements* 1.383.
[15] See below, III.48-80.

to realize that the verbal technique, tantamount to completing the square, always produces a correct solution.

The scope of the chapter on multiplication is limited to the multiplication of binomials by monomials and by binomials, the second term of a binomial being either positive or negative, the first term always positive. Al-Khwārizmī makes it clear that the first term of a binomial is in tens (*articuli*) and the second in units (*unitates*), and that if a binomial is multiplied by a binomial, four multiplications are required (each term of one by each term of the other) to reach the final product. He notes that if the second terms are both positive or negative, their product is added to the sum of the other partial products; if one is positive and the other negative, their product is taken from the sum. He begins with three specific examples – $(10 + 1)(10 + 2)$, $(10 - 1)(10 - 1)$, and $(10 + 2)(10 - 1)$ – which are worked out in detail. Since the student is presumed to know that each of these problems is only a reformation of familiar factors (11×12, 9×9, and 12×9), he is forced to accept the reasonableness of the rules for multiplying and adding negative numbers. Then he gives examples of binomials multiplied by a monomial – $(10 - x)\,10$ and $(10 + x)\,10$ – and works these out in detail. Thereafter follow nine examples, eight of which have an unknown in the binomial:

1. $(10 + x)(10 + x)$	6. $(10 + x)(x - 10)$
2. $(10 - x)(10 - x)$	7. $(10 + \frac{x}{2})(\frac{1}{2} - 5x)$
4. $(10 - x)(10 + x)$	8. $(10 + x)(x - 10)$
5. $(10 - x)x$	9. $(x + 10)(x - 10)$.

Repeating the sixth example in what I call the eighth, he varies the first factor to make the ninth example which he solves. The third example is interesting for its answer: $(1 - \frac{1}{6})(1 - \frac{1}{6}) = \frac{2}{3} + (\frac{1}{6} \cdot \frac{1}{6})$. He closes this section by repeating the rule that, if the second terms of the binomials are opposite in sign, their product is subtracted from the sum of the other partial products.

Three completed problems introduce the fifth chapter on computing with roots. Instead of explaining how these are solved as he did with solving equation, al-Khwārizmī proceeds immediately to methods for multiplying and dividing radical numbers. While he explains the techniques by examples, the steps in performing various operations are perhaps best displayed in modern generalizations:

(a) $a\sqrt{x^2} = \sqrt{a^2x^2} = ax$

(b) $a\sqrt{b^2} = \sqrt{a^2b^2} = \sqrt{c} = d$

(c) $\dfrac{\sqrt{a^2}}{\sqrt{b^2}} = \sqrt{\dfrac{a^2}{b^2}} = \dfrac{a}{b}$

(d) $\dfrac{a\sqrt{c^2}}{\sqrt{b^2}} = \sqrt{\dfrac{a^2c^2}{b^2}} = \dfrac{ac}{b}$

(e) $(\sqrt{a^2})(\sqrt{b^2}) = \sqrt{a^2b^2} = \sqrt{c} = d$

(f) $(a\sqrt{b^2})(c\sqrt{d^2}) = (\sqrt{a^2b^2})(\sqrt{c^2d^2}) = \sqrt{e} = f$.

For several of these he expects the student to recall how to find the square roots of numbers, whether they are perfect squares or not.

The last section on proofs (*cause*) offers intuitive explanations for the first two problems which introduce the fifth chapter. This is done by clever addition and subtraction of line segments set equal to the components of the left side of each problem:

$$(\sqrt{200} - 10) + (20 - \sqrt{200}) = 10$$
$$(20 - \sqrt{200}) - (\sqrt{200} - 10) = 30 - 2\sqrt{200}.$$

But the solution of the third problem,

$$[100 + (x^2 - 20x)] + [50 + (10x - 2x^2)] = 150 - (x^2 + 10x),$$

is offered verbally, much as it would be done today: similar terms on the left side of the equation are collected to yield the answer on the right. The verbal explanation was required because al-Khwārizmī knew of no way to combine line segments and geometric squares to produce the answer. In view of the verbal explanation, however, one may wonder why he did not sum up by remarking that the two previous problems solved by construction could be resolved easily, in so many words, by collecting like terms.

The discussion on radical numbers completes what may be called al-Khwārizmī's elementary theory of equations. The sixth chapter, on equations (*questiones*), poses six problems each illustrating a different type of equation, the techniques necessary to reduce each to its canonical form, and their respective solutions. The equations are:

(1) $x^2 = x(10 - x)4$

(2) $10^2 = 2\frac{7}{9}x^2$

(3) $\dfrac{10 - x}{x} = 4$

(4) $(\frac{x}{3} + 1)(\frac{x}{4} + 1) = 20$

(5) $(10 - x)^2 + x^2 = 58$

(6) $(\frac{x}{3})(\frac{x}{4}) = x + 24$.

Four technical words which describe operations necessary to put the problems into canonical forms appear in the solutions. They are:

(1) *reducere*: to reduce the coefficient of the squared term to unity by multiplying all terms of the equation by the reciprocal of the coefficient;
(2) *reintegrare*: the same as *reducere* except that the coefficient is less than unity;
(3) *opponere*: to subtract a positive term on one side of an equation from itself and from its like term on the other side;
(4) *restaurare*: to add the absolute value of a negative term from one side of an equation to itself and to the other side.

Twelve additional problems <VII. *Questiones varie*> reinforce much of what has preceded. They are a mixed bag containing a surprise. First, only four of the model equations receive further exemplification:

$ax^2 = c$: (example 9)
$bx = c$: (examples 2 and 7)
$ax^2 + bx = c$: (example 12)
$ax^2 + c = bx$: (examples 1, 2-6, 8, 10-11).

Secondly, the surprise is new material: fractional equations in examples (4), (5), (7), (8), (11), and (12). Two methods for solving these fractional equations are presented: first, the equivalent of cross-multiplication in (5), (7), (8), and (11); second, the equivalent of multiplying each term of the equation by the lowest common denominator in (4) and (12). Furthermore, example (7) requires the reader to readjust his thinking; the object of the problem or unknown is a square. Since the initial equation will eventually become a quadratic, al-Khwārizmī tells the student to treat it as *res*, the usual word for the first degree variable, *x*; otherwise, the problem produces a fourth degree equation which is outside the scope of the text.[16] Finally, the statements of the problems become these equations:

(1) $x(10-x) = 21$
(2) $(10-x)^2 - x^2 = 40$
(3) $(10-x)^2 + x^2 + (10-x) - x = 54$
(4) $\frac{10-x}{x} + \frac{x}{10-x} = 2\frac{1}{6}$
(5) $\frac{1}{2}\left(\frac{5x}{10-x}\right) = 5(10-x)$
(6) $(10-x)^2 = 81x$

[16] The technique of substituting *y* for x^2 is used extensively in *Liber augmenti et diminutionis*; see Libri, *Histoire des sciences mathématiques en Italie* 1.308 and passim.

(7) $x^2 = y, \frac{y}{y+2} = \frac{1}{2}$
(8) $\frac{x(10-x)}{(10-x)-x} = 5\frac{1}{4}$
(9) $(4x)(5x) = 2x^2 + 36$
(10) $(\frac{2}{3}x - 3)^2 = x$
(11) $\frac{3/2}{x+1} = 2x$
(12) $\frac{1}{x+1} = \frac{1}{x} - \frac{1}{6}$.

The last chapter in *Liber algebre* is a short section on proportion applied to business problems, the well-known 'Rule of Three'. Following clear statements about possible variations attendant upon three given numbers with the fourth to be found, i.e.,

$$\frac{a}{b} = \frac{c}{x} \quad \text{and} \quad \frac{a}{b} = \frac{x}{c},$$

two examples are worked through in detail. Interesting is the translator's use of the expression *numerus ignotus* for 'the number to be found', a phrase that does not appear in the theory of equations, as well as it might; there the unknown is always referred to as *res* or *census*. A third example closes the chapter; it was probably included for its practical value since it focuses upon payment for six days' work where the salary is set for one month's work. With this last problem the older manuscript copies of Gerard's translation conclude *Liber algebre*.

Found only in Gerard's translation are the contents of the Appendix on pp. 257-61 below. Robert of Chester's version shows an appendix that summarizes the rules for solving the six types of equations; William of Lunis offers, as though his own, nearly all of the algebraic section of chapter 15, part 3, of Fibonacci's *Liber abaci*; Rosen's Arabic source (MS. Oxford, Bodleian Library Hunt 214, fols. 1-34) includes three additional chapters, on mensuration, legacies, and computation of returns.[17] Gerard made it clear that he incorporated the material from another font, for he wrote: 'Liber hic finitur. In alio tamen libro repperi hec interposita suprascriptis' (below, Appendix 2). His statement certainly suggests that he recognized that the material was not written by al-Khwārizmī, yet he saw another copy of *al-Jabr* which contained the set of

[17] *The Algebra of Mohammed ben Musa* (London, 1831; rpt. New York, 1969), a frequent reference for Karpinski; see above, n. 5. Yet Rosen's translation has been severely criticized by J. Ruska, *Zur ältesten arabischen Algebra und Rechenkunst* (Sitzungsberichte der Heidelberger Akademie der Wissenschaften, phil.-hist. Klasse 8; Heidelberg, 1917). Toomer gives the locations for three of the Arabic manuscripts; see his 'al-Khwārizmī', *DSB* 7.364.

problems. Since one aspect of the value of the present critical edition is to appreciate a translation which from the sheer force of the number of extant copies is assumed to have provided a major thrust toward the development of algebra in medieval Europe, I judged it important to include the Appendix as a cognate part of al-Khwārizmī's tract, although I have no evidence that he was its author.

The Appendix is a selection of twenty-one problems making a very uneven group. About half the solutions are straightforward; the remainder do not come so easily. Early on, the student is confronted with three quartic equations in a row, (4)-(6), followed by a cubic. Although their solutions are shown to be similar to simple types studied before, the student does have to refine his tools for solving problems. Enough practice is offered, however, particularly for thinking of *census* in terms of *res* or *radix*.

Problems (15) and (19) are the most interesting, if not the most difficult. The former begins with the squares of two unknowns, instead of the customary 'Divide ten into two parts'. Two relationships are established between them, which permit a substitution from one equation into the other thereby reducing the problem to one equation in one unknown. In problem (19), for the first time, the student is confronted by a radical binomial in an equation. The wording of problem and solution, however, is obscure (the scribe's fault?); and the medieval Latin reader may have ignored this part as unintelligible. This is a pity, since a new technique lies here: squaring both sides of an equation to remove a radical term. Both problems are finally solved quite conventionally.

The problems in the Appendix may be expressed as follows:

(1) $(10 - x)^2 = 81$

(2) $10x = (10 - x)^2$

(3) $\frac{2}{3}(\frac{1}{5}x^2) = \frac{1}{7}x$

(4) $x^2(4x^2) = 20$

(5) $(x^2)(\frac{x^2}{3}) = 10$

(6) $(x^2)(4x^2) = \frac{x^2}{3}$

(7) $(x^2)x = 3x^2$

(8) $(3x)(4x) = x^2 + 44$

(9) $x(4x) = 3x^2 + 50$

(10) $x^2 + 20 = 12x$

(11) $(\frac{x^2}{3})(\frac{x^2}{4}) = x^2$

(12) $(x^2/$[illegible]$ + 1)(x^2/$[illegible]$ + 2) = x^2 + 13$

(13) $(x^2 - \frac{x^2}{3} - \frac{x^2}{4} - 4) = x^2 + 12$

(14) $x^2(\frac{2}{3}) = 5$

(15) $x^2 - y^2 = 2$ and $\frac{y^2}{x^2} = \frac{1}{2}$

(16) $x^2(3x) = 5x^2$

(17) $(x^2 - \frac{x^2}{3})3x = x^2$

(18) $\frac{1}{3}(x^2 - 4x) = 4x$ and $x^2 = 256$

(19) $\sqrt{x^2 - x} + x = 2$

(20) $(x^2 - 3x)^2 = x^2$

(21) $(x^2)(\frac{2}{3}x^2) = 5$.

THE LATIN MANUSCRIPTS

Manuscript copies of Gerard's translation begin 'Hic (*or* Sic) post laudem dei et ipsius exaltationem inquit (*or* inquid)' and generally conclude '... cuius radix est quinque'. They are easily separated into two groups. Most of the seven manuscripts in the first set are from the thirteenth and early fourteenth centuries and exhibit few significant variations among themselves. The second set of eight manuscripts are later in composition, show many variations from the first set and among themselves, offer fewer or more problems, and suggest that the terminology has been edited. The critical edition is based on the first group whose members are described below in detail; the various titles of the tract are given immediately after identification of the codices. The members of the second group are recognized as witnesses to the importance (or perhaps the availability) of Gerard's translation and are described in less detail. All and only scientific works in the codices of the first group are itemized; some works are marked with an asterisk to signal a translation to Gerard of Cremona. The diad TK following a title refers to Thorndike and Kibre's *Catalogue of Incipits*.[18]

Fonds of the Critical Edition

C = Cambridge, Cambridge University Library Mm.2.18, fols. 65rb-69vb ('Liber maumeti filii moysi alchoarismi de algebra et almuchabala incipit'). France, *c.* 1360.

[18] L. Thorndike and P. Kibre, *A Catalogue of Incipits of Mediaeval Scientific Writings in Latin*, 2nd edition (Cambridge, Mass., 1963).

Contents:

(1) fols. 2r-49r: Jabir ibn Aflaḥ al-Ishbīlī, *Flores de almagesto** [TK 1403]. (2) fols. 49r-65r: Anon., *Liber de numeris et lineis rationalibus** [TK 33]. (3) fols. 65rb-69vb: al-Khwārizmī, *Liber de algebra et almuchabala** [TK 624]. (4) fols. 69vb-76v: Abū Bakr al-Ḥasan ibn al-Khaṣīb, *De mensuratione terrarum** [TK 281]. (5) fols. 76v-77r: Abū ᶜUthmān Saᶜīd ibn Yaᶜqūb al-Dimashqī, *De mensuratione figurarum superficialium et corporearum** [TK 1390]. (6) fol. 77r: ᶜAbd al-Raḥmān, *De mensuratione** [TK 1387]. (7) fols. 77v-82r: Abraham ibn Ezra (?), *Liber augmenti et diminutionis* [TK 238].

The codex was commissioned by Geoffrey de Wighton, O.F.M., and paid for 'by alms given by his friends'.[19] Thomas Knyvett (d. 1622), Baron Escrick who discovered the gunpowder plot, obtained the book as his name and motto within testify. Thereafter it passed into the library of Thomas Moore (1646-1714). Upon his death the collection was purchased by King George I and presented to Cambridge University in 1715.[20] Items 2 through 7 may have been copied directly from Paris, Bibliothèque Nationale lat. 9335 or from its exemplar, since they are in exactly the same order as they appear in the Paris codex. The algebra, item 3, is the manuscript mentioned by Montfaucon. By and large, it is a very good copy with few variations from Paris lat. 9335, notably *kaficii* for *cafficii* (fol. 115rb) and only three omissions of significant length (fols. 113ra, 113va, 116rb), the first and third due to homoeoteleuton.

Bibliography: *A Catalogue of the Manuscripts Preserved in the Library of the University of Cambridge* 4 (Cambridge, 1861), pp. 132-38.

F = Florence, Biblioteca Nazionale Conv. soppr. J.V.18 (Codex S. Marci Florentini 216), fols. 80r-86v (no title). France/Italy, saec. XIII ex.

Contents:

(1) fols. 1r-2r: Anon., *Liber de umbris*. (3) fols. 4r-9v: Anon., *Liber ysoperimetrorum* [TK 1083 (3)]. (6) fols. 11r-12v: Anon., (*inc.*) 'Perisimetra sunt quorum latera coniunctim sunt...' [TK 1035]. (7) fols. 12v-16r: Anon., *Practica geometrie* [TK 870]. (8) fols. 17r-29v: Jordanus de Nemore, *De triangulis* [TK 760]. (9) fols. 30r-32r: Anon., *Liber de sinu demonstrato* [TK 477]. (10) fols. 33ra: Anon., *Quadratura per lunulas* [TK 1058]. (11) fols. 33rb: Thābit ibn Qurra, *De proportionibus* [TK 1139]. (12) fols. 33v-34r: Campanus de Novara, *De figura sectoris* [TK 280]. (13) fols. 37ra-39rb: Jordanus de Nemore, *Demonstratio in algorismum* (*inc.*: 'Numerorum alius simplex...') [TK 958]. (14) fols. 39rb-42va: Jordanus de Nemore, *Tractatus minutiarum* [TK 875]. (15) fols. 42v-53v: Jordanus de Nemore, *De numeris datis* [TK 959]. (16) fols. 53v-70r: John of Seville, *Algorismus* [TK 1250]. (17) fol. 70r-v: Anon., *Compotus*. (18) fols. 71r-72v: Robert Grosseteste, *De lineis angulis et figuris* [TK 1627]. (19) fols. 72v-80r: Anon., *De numeris fractis* [TK 1475]. (20) fols. 80r-86r: al-Khwārizmī, *Liber de algebra et almuchabala** [TK 624]. (21) fols. 87r-91v: Aḥmad ibn Yūsuf al-Kammād, *De proportione et proportionalitate* [TK 1006].

The algebra was obviously copied piecemeal by two scribes: the first was responsible for fols. 80r-81v and the second for fols. 82r-86v. The manuscript is significant for three reasons. First, although it contains more variants than the other three manuscripts used for the critical edition, its early date suggests a strong interest in al-Khwārizmī's algebra. Second, it is the only manuscript with the unusual spelling of *census*, namely, *sensus*, which occurs in the section copied by the first scribe. Third, a (near?) contemporary gloss attributes the translation incorrectly to William of Lunis: 'Incipit liber gebre de numero translatus a magistro Guillelmo de lunis in quadriviali sciencia peritissimo' (fol. 80ra). While the note is excellent testimony to the fact that William of Lunis did translate al-Khwārizmī's *al-Jabr*, it miscredits William with this translation. He is responsible for an entirely different translation which spawned its own family of copies.[21]

Bibliography: A. A. Björnbo, *Die mathematischen S. Marcohandschriften in Florenz*, 2nd edition, ed. G. C. Garfagnini (Quaderni di storia e critica della scienza, N.S.; Pisa, 1976), pp. 88-92; B. B. Hughes, ed. and trans., *Jordanus de Nemore. De numeris datis* (Publications of the Center for Medieval and Renaissance Studies 13; Berkeley, 1981), pp. 27-28; R. B. Thomson, 'Jordanus de Nemore: Opera', *Mediaeval Studies* 38 (1976) 97-144 passim.

M = Milan, Biblioteca Ambrosiana A 183 inf., fols. 115r-120r ('Incipit liber Mulumecti de algebra et almuchabila'). Northern Italy, saec. XIV in.

Contents:

(1) fol. 1r: Anon., *De compoto* (fragmentum finis). (2) fols. 1v-7r: John of Sacrobosco, *De spera* [TK 1577]. (3) fols. 7v-13v: Jordanus de Nemore, *De triangulis* [TK 260]. (4) fols. 14r-19v: Ptolemy, *Planispherium* [TK 1190]. (5) fols. 20v-21r: al-Bāttanī, (*excerptum inc.*) 'integrorum multiplicantis'. (6) fols. 22r-23r: Campanus de Novara, *Almanach coniunctionum mediarum solis et lune*. (7) fols. 24r-28v: al-Qabīsī (trans. John of Seville), *Ad iudicia astrorum* (fragmentum). (8) fols. 29r-56r: Sahl ibn Bishr, *De significatione temporis ad iudicia* [TK 1411]. (9) fols. 56v-64v: Māshā'allāh (trans. John of Seville), *De receptionibus* [TK 774]. (10) fols. 64v-68v: Māshā'allāh (trans. John of Seville), *De revolutione annorum mundi* [TK 362]. (11) fols. 68v-71r: Māshā'allāh, *De coniunctionibus planetarum* [TK 729]. (12) fols. 71r-73r: Pseudo-Hippocrates, *Liber astronomiae*. (13) fols. 74r-76r: Thābit ibn Qurra, *Imagines* [TK 285]. (14) fols. 76r-77v: Thābit ibn Qurra, *Super almagestum* [TK 1570]. (15) fols. 77v-78v: Thābit ibn Qurra, *De motu octave spere* [TK 661]. (16) fol. 79r: Anon., *Brevis tractatus de sperico*

[19] 'Iste liber est Fratris Galfridi de Wyghtone quem fecit scribi de elemosinis amicorum suorum' (fol. 1r). See A. B. Emden, *A Biographical Register of the University of Oxford to A.D. 1500* 3 (Oxford, 1959), p. 2045. The codex is not mentioned by N. R. Ker, *Medieval Libraries of Great Britain*, 2nd edition (London, 1964), even though Friar Geoffrey lived and died in England.

[20] Many of the details here were graciously supplied by Jayne Cook, Assistant Under-Librarian of Cambridge University Library, to whom my thanks.

[21] See n. 6 above.

corpore et solido. (17) fols. 80-114 desunt. (18) fols. 115r-120r: al-Khwārizmī, *Liber de algebra et almuchabala** [TK 624]. (19) fols. 120v-122v: Anon., *Algorismus* [TK 990].

A noteworthy gathering of treatises copied by several Italian (and French?) scribes from the mid-thirteenth to the mid-fourteenth century, the codex is witness to a strong interest in scientific topics. The copy of *Liber algebre* displays differences in notation as well as improvements upon explanations. For instance, to represent 3 1/3 the scribe wrote 1/3 3. For textual emendation, in place of 'Dic: "Hic ... equales rei"' (below, VII.102-104), there appears this clearer statement: 'Pro minori censu pone rem. Pro maiori vero censu pone rem et duas dragmas. Quibus multiplicatis per mediam dragmam que provenit ex divisione minoris censi (*sic*) per maiorem et eveniunt media res et dragmam (*sic*), id est, que equantur uni rei' (fol. 118va27-34). Seemingly, early fourteenth-century scholars were seeking clarifications and improvements upon texts handed them. Yet the copy has numerous defects; a representative selection are shown in the apparatus.

Bibliography: P. Revelli, *I codici ambrosiani di contenuto geografico* (Milan, 1929), pp. 24-25; A. L. Gabriel, *A Summary Catalogue of Microfilms of One Thousand Scientific Manuscripts in the Ambrosiana Library, Milan* (Notre Dame, Ind., 1968), p. 44.

N = Paris, Bibliothèque Nationale fr. 16965, fols. 2r-19v ('Liber mahumeti filii moysi alchorismi de algebra et almuchabla incipit'). France, saec. XVI in.

Contents:

(1) fols. 2r-19v: al-Khwārizmī, *Liber de algebra et almuchabala** [TK 624]. (2) fols. 20r-26v: Anon., *Excerptio uel Expositio compoti Herici*. (10) fols. 379r-407r: Rudolph of Spoleto, *De proportione proportionum disputatio*. (11) fols. 408r-448r: Anon., *Arithmetica logarithmica*.

Sometime in the Saint-Germain collection, the codex is an anthology of scientific works copied in the sixteenth and seventeenth centuries, both in Latin and in French (twelve titles not identified above). The algebra was written in a very clear humanistic hand, most probably copied from Paris, Bibliothèque Nationale lat. 9335, and at one time the manuscript was part of the Lustierine Library. There are no significant variants to recommend its use for the critical edition.

Bibliography: L. Delisle, *Inventaire général et méthodique des manuscrits français de la Bibliothèque Nationale* 2 (Paris, 1876; rpt. 1975), p. 235.

P = Paris, Bibliothèque Nationale lat. 9335, fols. 110vb-116va ('Liber maumeti filii moysi alchoarismi de algebra et almuchabala incipit'). Southern France/Italy, saec. XIII in.

Contents:

(1) fols. 1r-19r: Theodosius of Bithynia, *De speris** [TK 1523]. (2) fols. 19r-21v: Autolycus of Pitane, *De motu spere** [TK 1151]. (3) fols. 22r-23r: Asculeus, *De ascensione signorum** [TK 1449]. (4) fol. 23v: Anon., (*inc.*) 'Cordam per archum et

archum per cordam invenire'. (5) fols. 23v-25r: Thābit ibn Qurra, *Introductio in almagestum* [TK 502]. (6) fols. 25r-28v: Theodosius of Bithynia, *De locis habitabilibus* [TK 684]. (7) fol. 28v: Anon., *Ordo qui est post librum euclidis secundum quod invenitur in scriptis Iohanicii*. (8) fols. 28v-30r: *Liber Arsamitis de mensura circuli*. (9) fols. 30r-31v: Ahmad ibn Yūsuf, *De arcubus similibus* [TK 624]. (10) fols. 31v-32v: al-Kindī, *De quinque essentiis** [TK 1376]. (11) fols. 32v-54v: Menelaos, *De figuris spericis** [TK 397]. (12) fols. 55v-63r: Banū Mūsa, *Liber trium fratrum* [TK 832]. (13) fols. 63v-64v: Anon., (*inc.*) 'Iste modus est sufficiens in arte eptagoni cadentis in circulo'. (14) fols. 64v-75r: Ahmad ibn Yūsuf, *De proportione et proportionalitate* [TK 1139]. (15) fols. 75r-82r: al-Kindī, *De aspectibus* [TK 1013]. (16) fols. 82r-83v: Pseudo-Euclid, *De speculis* [TK 1084]. (17) fols. 84r-88v: al-Kindī, *De speculis* [TK 1388]. (18) fols. 88v-92r: Anon., *De aspectibus euclidis*. (19) fols. 92v-110r: Muḥammad ibn ᶜAbd al-Bāqī al-Baghdadī, *Commentaria in euclidis elementis lib. X** [TK 333]. (20) fols. 110vb-116va: al-Khwārizmī, *Liber de algebra et almuchabala** [TK 624]. (21) fols. 116v-125v: Abū Bakr al-Ḥasan ibn al-Khaṣīb, *De mensuratione terrarum** [TK 281]. (22) fols. 125v-126r: Abū ᶜUthmān Saᶜrd ibn Yaᶜqūb al-Dimashqī, *De mensuratione figurarum* [TK 1390]. (23) fol. 126r-v: Aderametus (ᶜAbd al-Raḥmān?), *De mensuratione* [TK 1387]. (24) fols. 126v-133v: Abraham ibn Ezra (?), *Liber augmenti et diminutionis* [TK 238]. (25) fols. 135r-139v: al-Kindī, *De gradibus medicine* [TK 1228]. (26) fols. 140r-141r: Anon., *Capitulum cognitionis mansionis lune*. (27) fols. 141r-143r: Thābit ibn Qurra, *In motus accessionis et recessionis*. (28) fols. 143v-151v: al-Farābī, *De scienciis** [TK 925]. (29) fols. 151v-160v: Harib ibn Zeid, *De hortis et plantationibus*.

Recognized as 'perhaps the most important manuscript of Gerard of Cremona's works',[22] the codex is a veritable mine of medieval resources, twenty-nine tracts in pure and applied mathematics. The 161 leaves are of parchment, the two columns of text were written by a single hand, initials are red and blue, and an early table of contents appears on fol. 1. The algebra, item 20, is obviously the best text of all the manuscripts reviewed: the wording is unambiguous, the diagrams are helpfully complete, and the marginalia evince careful corrections by the same scribe who penned the text. (I incorporated these corrections as well as others made by him, interlinear or over-penned, into the text of the critical edition and noted them in italics). These features, reinforced by the manuscript's having been copied within perhaps fifty years of the translation and, conjecturally, from the final draft of Gerard, make P the exemplar for all copies in its genre.

Bibliography: L. Delisle, *Inventaire des manuscrits conservés à la Bibliothèque Impériale sous les n. 8823-11503 du fonds latin* (Paris, 1863), no. 9335; A. A. Björnbo, 'Über zwei mathematischen Handschriften aus dem vierzehnten Jahrhundert', *Bibliotheca mathematica*, 3rd Ser., 3 (1902) 63-75 and corrections to this by Björnbo in 'Handschriftenbeschreibung', *Abhandlungen zur Geschichte der mathematischen Wissenschaften mit Einschluss ihrer Anwendungen* 26.1 (1910) 138.

[22] R. H. Rouse, 'Manuscripts Belonging to Richard de Fournival', *Revue d'histoire des textes* 3 (1973) 256-57.

Q = Paris, Bibliothèque Nationale lat. 7377A, fols. 34r-43v ('Liber maumeti filii moysi alchoariximi de algebra et almuchabala incipit'). France, saec. XIII.

Contents:

(1) fols. 1r-33v: Anon., *Commentarius in decimum euclidis librum*. (2) fols. 34r-43v: al-Khwārizmī, *Liber de algebra et almuchabala** [TK 624]. (3) fols. 43v-58v: Abū Bakr al-Ḥasan ibn al-Khaṣīb, *De mensuratione terrarum** [TK 281]. (4) fols. 58v-70v (?): Abraham ibn Ezra (?), *Liber augmenti et diminutionis* [TK 238]. (5) fols. 71v-97v: Anon., *Scholium de mensuratione pentagoni et decagoni*. (6) fols. 99r-208r: Anon., *Tractatus de arithmetica*.

The *Liber de algebra* was copied directly from Paris lat. 9335 (P) during the third quarter of the thirteenth century, possibly by a Parisian university scribe, and the codex was sometime in the Colbertine Library. Not only are there comparatively few variations between the two manuscripts, but the corrections found in the margins of Paris lat. 9335 were often copied onto the same relative places in Paris lat. 7377A. Two noteworthy variations in spelling occur: *kaficii* for *cafficii* and *centexime* for *centessime*. This copy has nothing to offer the critical edition.

Bibliography: *Catalogus codicum manuscriptorum Bibliothecae Regiae* 4 (Paris, 1744), p. 349.

V = Vatican City, Biblioteca Apostolica Vaticana Vat. lat. 5733, fols. 275r-287r ('Incipit liber Mahumed filii Moysi Algorismi de algebra et almutabala transcriptus a magistro Simone Cremonensi in Toleto de arabico in latinum'). Italy, saec. XVI in.

Contents:

(4) fols. 189r-195r: Hermes, *Liber de quindecim stellis* [TK 768]. (7) fols. 211r-229v: Averroes (trans. Cal. Calonymus), *Destructio destructionis*. (12) fols. 275r-287r: al-Khwārizmī, *Liber de algebra et almuchabala** [TK 624].

This collection of scientific tracts centers on the work of Petrus Pomponatius of Mantua (1462-1524) and was sometime part of the library of the gymnasium at Bologna. In general, the algebra text is reliable as far as it goes; but it ends with the last of the *Questiones varie*, without the section on proportion nor with the set of extra problems. On a separate folio (274r) is a unique title for the tract, *Ars algebrae*, which begins on fol. 275r along with the completely erroneous ascription of the translation to 'magistro Simone Cremonensi'. The body of the text is well-written in a simple cursive hand and is divided into clearly stated sections, much as I have done, with subtitles. The only addition of any significance to the text is an insertion carefully placed within parentheses (fol. 282r) which I have included in the critical apparatus at VI.18 below. While a good witness to the continuing interest in the work of al-Khwārizmī, this manuscript has nothing further to add to the critical edition.

Bibliography: R. Lemay, ed., *Petri Pomponatii Mantuani Libri quinque de fato, de libero arbitrio et de praedestinatione* (Lucca, [1957]), pp. xxxi-xxxiii.

A comparison of variants between the texts of the first group of manuscripts produces ten readings whereby the manuscripts can be separated into two families, α and β, namely:

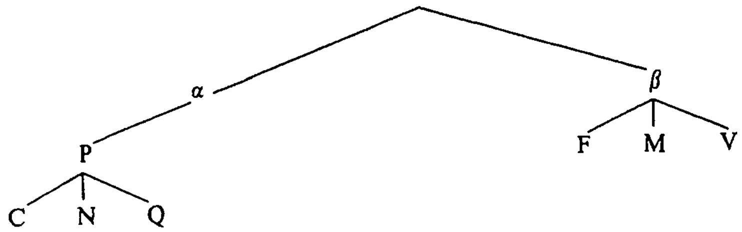
V-1: comprehendi potest de numeris ultime (I.13-14)
V-2: ad infinitam numerorum comprehensionem (I.13-14)
V-3: questio est impossibilis (II.B.55)
V-4: questio est destructa (*or* destracta) (II.B.55)
V-5: dupla ergo radicem novem (V.37-38)
V-6: multiplica ergo radicem novem (V.37-38)
V-7: capitula numerationis et eorum modos (VI.2)
V-8: capitula et eorum modos (VI.2)
V-9: reintegres censum tuum (VII.142-143)
V-10: reintegres novem radices (VII.142-143).

V-1 and V-2 are different conclusions for the first paragraph of the tract; each of the seven manuscripts shows either one or the other expression. V-3 and V-4 are different though meaningfully the same description of the possibility of solving a particular type of an equation. Six of the manuscripts have one or the other reading; the Vatican manuscript lacks the clause in a long passage omitted possibly because of homoeoteleuton. Again, with respect to V-5 and V-6, all manuscripts have either one or the other variant reading. The same holds for V-7 and V-8, and for V-9 and V-10. The manuscripts with their respective variants can be displayed in a matrix:

Manuscripts		V-1	V-2	V-3	V-4	V-5	V-6	V-7	V-8	V-9	V-10
Cambridge	(C)	x		x		x		x		x	
Paris fr. 16965	(N)	x		x		x		x		x	
Paris lat. 9335	(P)	x		x		x		x		x	
Paris lat. 7377A	(Q)	x		x		x		x		x	
Florence	(F)		x		x		x		x		x
Milan	(M)		x		x				x		x
Vatican	(V)		x				x		x		x

The pattern recommends a clear division of the manuscripts into two families. Hence, the members of the α family are C, N, P, and Q. Members of the β family are F, M, and V.

Regarding intrafamilial relationships: in the α family P is clearly the oldest in the group and the best extant copy. Q is a nearly perfect reproduction of P, with only one omission of any length (below, Appendix 115-116). N seems to be a better copy of P, for it has the passage missing in Q. C has its own set of omissions which are not found in N, P, or Q, and it does contain the sentence missing in Q. These factors make a case for the genealogy shown below for the members of the α family. The β family consists of three strikingly different manuscripts. They may be briefly described as exhibitors of at least one major characteristic unique to each. F explains the word 'algebra' by *oppositio* and 'almuchabala' by *responsio*. M has a unique substitution at VII.102-104. V would have us believe from the title that the tract was translated by Simon of Cremona. Hence, none is a direct descendant of either of the other two members of the β family. A stemma sets forth the relationships:



Manuscript Witnesses

The eight manuscripts briefly described below witness to efforts of scholars to improve upon Gerard's translation. Each offers modifications in terminology, addition or omission of problems, or additional textual material. Hence, none of these manuscripts was used to construct the critical edition. They merit attention, however, as witnesses to the importance of Gerard's translation which served as their foundation, not to overlook the burgeoning interest in algebra.

(1) Berlin, Deutsche Staatsbibliothek Hamilton 692, fols. 279r-291v ('In nomine dei eterni. Incipit liber Mauchumeti in Algebra et Almuchabula qui est origo et fundamentum totius scientie arismetice'). Italy, saec. xvi in.

inc.: Hic post laudem dei et ipsius exaltationem inquit...
expl.: ... et proueniunt 25, cuius radix est 5.

In general this copy has the characteristic variants of the β family. It is classified as a witness because of the numerous subtitles which were added to the text. Furthermore it adds six problems to the *Questiones varie* and omits problems 1, 14, and 15 from the Appendix.

Bibliography: H. Boese, *Die lateinischen Handschriften der Sammlung Hamilton zu Berlin* (Wiesbaden, 1966), pp. 334-35.

(2) Berlin, Staatsbibliothek Preussischer Kulturbesitz Lat. qu. 529, fols. 2r-16v ('... Macumetii ... Algebra...'). Italy, saec. xv med.

inc.: <H> ic post laudem dei et ipsius exaltationem inquit...
expl.: ... et proueniunt 25 cuius radix est quinque.

Despite considerable water damage (humidity?) to the outer edges of the leaves, much of the text can be read. It is clearly a member of the β family, but several times removed. There are a number of editorial changes in the text ans problems 1, 14, and 15 are missing from the Appendix.

Bibliography: E. Narducci, *Catalogo di manoscritti ora posseduti da D. Baldassarre Boncompagni*, 2nd edition (Rome, 1892), p. 106 n. 179 (no. 265).[23]

(3) Madrid, Biblioteca Nacional 9119 (olim Aa 30), fols. 352v-360v ('Incipit liber Mavmet filii Moysi Algorismi de Algebra et Almuchabala: Translatus a Magistro Gerardo Cremonensi in Toleto: de arabico in latinum'). Italy, saec. xv ex.

inc.: Hic post laudem dei et ipsius exaltationem inquid...
expl.: ... provenit .25. dragme cuius radix est .5.

While the text has many characteristics typical of the β family, it also evinces numerous editorial changes, some of which are simply erroneous. Furthermore, the text does not end with the twenty-first supplementary problem but continues on (fols. 360v12-363v) with sixteen additional algebraic problems, twenty-eight definitions for arithmetic and geometry, a tract on extraction of roots, and eighteen problems and rules for geometry and astronomy. Only a large *Q* in the margin introducing the word *Quod* signals the beginning of this second appendix.

Bibliography: J. L. Heiberg, 'Neue Studien zu Archimedes', *Zeitschrift für Mathematik und Physik* (Supplement 1890) 5; M. Clagett, *Archimedes in the Middle Ages*, vol. 2.1-2: *The Translations from the Greek by William of Moerbeke* (Philadelphia, 1976), pp. 69-71.

(4) Milan, Biblioteca Ambrosiana P 81 sup. (*olim* YS), fols. 1r-22r ('Machumeti de Algebra et Almuchabala, id est recuperationis et oppositionis. Liber incipit'). Italy, saec. xv in.

inc.: <H> ic post laudem dei et ipsius exaltationem inquid...
expl.: ... provenit radix de xxv et illa est v. Et cetera.

This copy is a member of a family of four displaying the same unique characteristics, notably, the frequent use of *cosa* for *res* and the addition of two lengthy paragraphs which begin 'Modus dividendi'. As a matter of convenience, I identify the set as 'the Modus family'. Also characteristic of the family is the sentence 'Sed ut res gravis leuis tibi fiat, sequatur id quod ex questionibus in textu propinquis cum ille erat per quorum significationem in aliis consimiliter operaberis, si deus voluerit' (fol. 11r). The codex was one time the property of Gian Vincenzo Pinelli (1535-1601).

[23] The manuscript is not recorded by V. Rose and F. Schillmann, *Verzeichnis der lateinischen Handschriften der Königlichen Bibliothek zu Berlin*, 3 vols. (Berlin, 1893-1919). However, the microfilm shows two parts (what appears to be the first and last third) of a label: [B. B 1 ... agni 9] The obvious deduction led me to Narducci's catalogue whose description fits exactly the contents of the manuscript shown on the microfilm.

Bibliography: A. Rivolta, *Catalogo dei codici pinelliani dell' Ambrosiana* (Milan, 1933), p. 40; A. L. Gabriel, *A Summary Catalogue of Microfilms of One Thousand Scientific Manuscripts in the Ambrosiana Library, Milan* (Notre Dame, Ind., 1968), p. 307.

(5) New York, Columbia University, Butler Library Plimpton 188, fols. 73r-82v ('Liber Mahucmeti de Algebra et Almuchabala id est recuperationis et oppositionis'). German hand, 1456.

inc.: Hic post laudem dei et ipsius exaltationem inquit...
expl.: ... provenit radix de 25 et illa est 5. Et cetera.

The text clearly belongs to the Modus family. Apart from a single space of one line, there is nothing to signal the end of al-Khwārizmī's tract and the beginning of what I would call *collectanea mathematica*: a miscellany of problems solved verbally and symbolically (fols. 82v-84v), a precis of al-Khwārizmī's algebra (fols. 85r-88r), two more problems solved symbolically (fols. 88v-89r), and a set of notes on arithmetic, algebra, and geometry (fols. 90r-94r).

Bibliography: S. De Ricci and W. J. Wilson, *Census of Medieval and Renaissance Manuscripts in the United States and Canada* 2 (New York, 1937; rpt. 1961), pp. 1787-88; D. E. Smith, *Rara arithmetica. A Catalogue of the Arithmetics Written before the Year MDCI, with a Description of Those in the Library of George Arthur Plimpton of New York* (Boston-London, 1908), pp. 454-56, 468, 480, 486-87 (Smith's description of the contents of the codex differs considerably from the undated, typed analysis which accompanied the microfilm).

(6) Paris, Bibliothèque Nationale ital. 949, fols. 226r-247v ('Incipit liber muchumeti de algebra et almuchabala recuperationis et opperationis'). Italy, 11 December 1450.

inc.: Hic post laudem dei et ipsius exaltationem...
expl.: ... et prouenient 25 dragme cuius radix est 5.

A remote member of the β family, the manuscript shows interesting marginalia in the hand of the scribe; for instance, next to the term 'medietas census et 5 radices equantur 28', the reader sees '. c . ra . dg .' under which lies '. 1 . 10 . 56 .'. These suggest an attempt at abbreviation as well as the procedure for changing the coefficient of the second degree term to unity (fol. 227v). Nonetheless the copy has a number of flaws, particularly the omission of problems 1, 14, and 15 from the Appendix.

Bibliography: G. Mazzatinti, *Inventario dei manoscritti italiani delle biblioteche di Francia* 1 (Rome, 1886), p. 169.

(7) Turin, Biblioteca Nazionale Universitaria H V 45, fols. 1r-36r ('Machvmeti de Algebra et Elmvchabala id est de recuperatione et oppositione'). Saec. XVI ex.

inc.: Sic post laudem dei et ipsius exaltationem, quam ad computationem consideravi necessarium...
expl.: ... provenit radix de 25. et illa est quinque.

The copy is a member of the Modus family and is the only item in the codex. Its interest lies in the beautiful cursive hand of the scribe and the large number of errors which must have proved frustrating to any average student; the errors begin with the initial capital of the *incipit*.

Bibliography: B. de Montfaucon, *Bibliotheca bibliothecarum manuscriptorum nova* 2 (Paris, 1739), p. 1399e. See also *Index alphabétique des livres qui si trouvent en la Bibliothèque Royale de Turin en cette année 1713* (now MS. R I 5 in the Biblioteca Nazionale Universitaria, Turin), p. 619.

(8) Vatican City, Biblioteca Apostolica Vaticana Urb. lat. 1329, fols. 43r-63r ('Machumeti de algebra et almuchabala, id est recuperationis et opposicionis'). Rome, 23 October 1458.

inc.: Sic post laudem dei et ipsius exaltationem inquit...
expl.: ... pervenit radix de xxv et illa est v. Et cetera.

While this copy is a member of the Modus family, two items make it special apart from the initial error in the *incipit*. First, and for the earliest time I have found the expression to be used, the text contains the words 'Probacio huius satis pulchra' (fol. 57v). The beautiful proof, however, turns out to be nothing more than a clever manipulation of numbers to produce a desired root. The second is the use of Roman numerals to write fractions; for instance, $2\frac{1}{4}$ is written II$\frac{\text{I}}{\text{IIII}}$. There are changes in wording of some problems and the entire section, *Capitulum conventionum negociatorum*, is omitted.

Bibliography: C. Stornajolo, *Codices Urbinates latini* 3 (Rome, 1921), pp. 268-69; W. Van Egmond, *The Commercial Revolution and the Beginnings of Western Mathematics in Renaissance Florence, 1300-1500* (Diss. Michigan, 1977), pp. 510-11.

The Critical Edition

Gerard of Cremona's translation of al-Khwārizmī's *al-Jabr*, as transmitted in P, fols. 110v-116v, follows. Corrections by the scribe are included in the text of the critical edition and are printed in italics. Significant variations from P are noted, as they appear in C, F, and M. The apparatus also contains more than eighty variant readings, additions, or omissions found in Libri's edition (L).[24] Contractions and abbreviations have been expanded according to conventional usage. The orthography of P is preserved except that *u* is used for *v*. Numbers in P are written as words, and this feature has been retained. I have supplied many paragraph divisions and subtitles as would benefit the sense of the text.

[24] See n. 3 above; Libri's edition was made from the manuscripts I have here designated NPQ.

Sigla

C Cambridge, Cambridge University Library Mm.2.18
F Florence, Biblioteca Nazionale Conv. soppr. J.V.18
M Milan, Biblioteca Ambrosiana A 183 inf.
P Paris, Bibliothèque Nationale lat. 9335

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L G. Libri, *Histoire des sciences mathématiques en Italie depuis la renaissance des lettres jusqu'à la fin du XVII[e] siècle* 1 (Paris, 1838), pp. 253-97 (edition of Gerard of Cremona's translation).

(P 110vb) LIBER MAUMETI FILII MOYSI ALCHOARISMI DE ALGEBRA ET ALMUCHABALA INCIPIT

< I. De numeris decimalibus et algebraicis >

Hic post laudem dei et ipsius exaltationem inquit: Postquam illud quod ad computationem est necessarium consideravi, repperi totum illud numerum fore, omnemque numerum ab uno compositum esse inveni. Unus itaque inter omnem consistit numerum. Et inveni omne quod ex numeris verbis exprimitur esse quod unus usque ad decem pertransit. Decem quoque ab uno progreditur, qui postea duplicatus et triplicatus et cetera quemadmodum fit de uno. Fiunt ex eo viginti et triginta et ceteri usque quo compleatur centum. Deinde duplicatur centum et triplicatur quemadmodum ex decem, et fiunt ex eo ducenta et trecenta, et sic usque ad mille. Post hoc similiter reiteratur mille apud unumquemque articulum usque ad id quod comprehendi potest de numeris ultime.

Deinde repperi numeros qui sunt necessarii in computatione algebre et almuchabale secundum tres modos fore, qui sunt: radicum et census et numeri simplicis non relati ad radicem neque ad censum. Radix vero que est unum eorum est quicquid in se multiplicatur ab uno, et quod est super ipsum ex numeris, et quod est preter eum ex fractionibus. Census autem est quicquid aggregatur ex radice in se multiplicata. Sed numerus simplex est quicquid ex numeris verbis exprimitur absque proportione eius ad radicem et ad censum.

< II. De modis equationum >

< A. Tres modi simplices >

Ex his igitur tribus modis sunt qui se ad invicem equant. Quod est sicut si dicas: 'Census equatur radicibus, et census equatur numero, et radices equantur numero.' Census autem qui radicibus equatur est ac si dicas: 'Census equatur quinque radicibus.' Radix ergo census est quinque. Et census est viginti

1-2 *om. F, sed manus coaeva add. in calce*: Incipit liber gebre de numero translatus a magistro Guillelmo de lunis in quadriviali sciencia peritissimo

I 5 est] esse *F* 7 inveni *om. F* 11 ex[1] *om. F* 12 sic] cetera *F* similiter] simile *M* 13-14 ad ... ultime] ad infinitam numerorum comprehensionem *FM* 15-16 in ... almuchabale] in computatione oppositionis algebre et responsionis almuchabale *F* 16 radicum] radicis *F*: radices *M* 17 neque] usque *CF* unum] unus *L* 20 simplex] sensus *F*

II.A 3 equant] equantur *L* quod] qui *M* 5 census[1]] sensus *F* (*et saepe infra*) quinque. Ipse namque quinque suis radicibus equalis existit. Et sicut si dicas: 'Tertia census equatur quattuor radicibus.' Totus igitur census est duodecim radices qui est centum quadraginta quattuor. Et sicut si dicas: 'Quinque census equantur decem radicibus.' Unus igitur census duabus equatur radicibus. Ergo radix census est *duo, et census* est quattuor. Similiter quoque quod fuerit maius censu aut minus, ad unum reducetur censum. Et eodem modo fit ex eo quod ipsi equatur ex radicibus. Census autem qui numero equatur est sicut cum dicitur: 'Census equatur novem.' Ipse igitur est census et radix eius est tres. Et sicut si dicas: 'Quinque census equantur octoginta.' Unus igitur census est quinta octoginta qui est sedecim. Et sicut si dicas: 'Medietas census equatur decem et octo.' Ergo census equatur triginta sex. Et similiter omnis census augmentatus et diminutus ad unum reducitur censum. Et eodem modo fit de eo quod ei equatur ex numeris. Radices vero que numeris equantur sunt sicut si dicas: 'Radix equatur tribus.' Radix est tres. Et census qui est ex ea est novem. Et sicut si dicas: 'Quattuor radices equantur viginti.' Una igitur ra(P 111ra)dix equatur quinque. Et similiter sic dicas: 'Medietas radicis equatur decem.' Ergo radix est viginti. Et census qui est ex ea est quadringenta.

<B. TRES MODI COMPOSITI>

Hos preterea tres modos qui sunt radices et census et numerus inveni componi. Et sunt ex eis tria genera composita, que sunt hec: census namque et radices equantur numero; et census et numerus equantur radicibus; et radices et numerus equantur censui. Census autem et radices que numero equantur sunt sicut si dicas: 'Census et decem radices equantur triginta novem dragmis.' Cuius hec est significatio: ex quo censu cui additur equale decem radicum eius aggregatur totum quod est triginta novem. Cuius regula est ut medies radices que in hac questione sunt quinque. Multiplica igitur eas in se et fiunt ex eis viginti quinque. Quos triginta novem adde, et erunt sexaginta quattuor. Cuius radicem accipias que est octo. Deinde minue ex ea medietatem radicum que est quinque. Remanet igitur tres qui est radix census. Et census est novem. Et si duo census aut tres aut plures aut pauciores nominentur, similiter reduc eos ad censum unum. Et quod ex radicibus aut numeris est cum eis, reduc ad similitudinem ejus ad quod reduxisti censum. Quod est ut dicas: 'Duo census et

10 unus] unde *M* 13 ipsi ... radicibus] equantur censui ipsi radicibus *F* ex *om. F* 16 *post* quinta *add.* de *F* 16-17 medietas ... octo] quinque census equantur 180 *M* 17 et[1] *om. L* 20 radix[1] ... novem *om. M*

II.B 2 *post* modos *add.* simplices *F* 3 et sunt *om. F* sunt[1]] sicut *L* 6 triginta] decem et *C* 7 quo] quolibet *F* censu] censuum *M* 10 quos] quibus *F* 12 et census *om. C* 13 similiter] semper *F* 14 aut *om. F* est] et *L* ad *om. F*

decem radices equantur quadraginta octo.' Cuius est significatio quod cum quibuslibet duobus censibus additur equale decem radicum unius eorum, aggregantur inde quadraginta octo. Oportet itaque ut duo census ad unum reducantur censum. Novimus autem iam quod unus census duorum censuum est medietas. Reduc itaque quicquid est in questione ad medietatem sui. Et est sicut si dicatur: 'Census et quinque radices equales sunt viginti quattuor.' Cuius est intentio quod cum cuilibet censui quinque ipsius radices adduntur, aggregantur inde viginti quattuor. Media igitur radices et sunt duo et semis. Multiplica ergo eas in se et fient sex et quarta. Adde hoc viginti quattuor et erunt triginta et quarta. Cuius accipias radicem que est quinque et semis. Ex qua minue radicum medietatem que est duo et semis. Remanet ergo tres qui est radix census, et census est novem.

Et si dicatur: 'Medietas census et quinque radices equantur viginti octo.' Cuius quidem intentio est quod cum cuiuslibet census medietati additur equale quinque radicibus ipsius, proveniunt inde viginti octo. Tu autem vis ut rem tuam reintegres donec ex ea unus proveniat census. Quod est ut ipsam duplices. Duplica ergo ipsam et duplica quod est cum ea ex eo quod equatur ei. Erit itaque quod census et decem radices equantur quinquaginta sex. Media ergo radices, et erunt quinque. Et multiplica eas in se et provenient viginti quinque. Adde autem eas quinquaginta sex et fient octoginta unum. Cuius accipias radicem que est novem. Et minuas ex ea medietatem radicum que est quinque. Et remanent quattuor qui est radix census quem voluisti. Et census est sedecim cuius medi(P 111rb)etas est octo. Et similiter facias de unoquoque censuum, et de eo quod equat ipsum ex radicibus et numeris.

Census vero et numerus qui radicibus equantur sunt sicut si dicas: 'Census et viginti una dragma equantur decem radicibus.' Cuius significatio est quod cum cuilibet censui addideris viginti unum, erit quod aggregabitur equale decem radicibus illius census. Cuius regula est ut medies radices et erunt quinque. Quas in se multiplica et proveniet viginti quinque. Ex eo itaque minue viginti unum quem cum censu nominasti et remanebit quattuor. Cuius accipies radicem que est duo. Quam ex radicum medietate, que est quinque, minue. Remanebit ergo tres qui est radix census quem voluisti; et census est novem. Quod si volueris, addes ipsam medietati radicum et erit septem. Qui est radix census; et census est quadraginta novem. Cum ergo questio evenerit tibi deducens te ad hoc capitulum, ipsius veritatem cum additione experire. Quod si non fuerit, tunc procul dubio erit cum diminutione. Et hoc quidem unum trium

21 *post* radices *add.* sunt *F* 30 proveniunt] perveniunt *L* (*et saepe infra*). 30-31 rem tuam] cum *F* 32 quod equatur] quo equatur *F* 45 quem] quam *L* accipies] accipiam *F* 48 *post* ipsam *add.* radicem que est 2 *F* 51 tunc *om. F*

capitulorum in quibus radicum mediatio est necessaria progreditur cum additione et diminutione. Scias autem quod cum medias radices in hoc capitulo et multiplicas eas in se, et fit illud quod aggregatur minus dragmis que sunt cum censu, tunc questio est impossibilis. Quod si fuerit eisdem dragmis equalis, tunc radix census est equalis medietati radicum absque augmento et diminutione. Et omne quod tibi evenerit ex duobus censibus aut pluribus aut paucioribus uno censu, reduc ipsum ad censum unum sicut est illud quod in primo ostendimus capitulo.

Radices vero et numerus que censui equantur sunt sicut si dicas: 'Tres radices et quattuor ex numeris equantur censui uni.' Cuius regula est *ut* medies radices que erant unus et semis. Multiplica ergo ipsas in se, et provenient ex eis duo et quarta. Ipsum itaque quattuor dragmis adde et fiunt sex et quarta. Cuius radicem que est duo et semis assume; quam medietati radicum que est unus et semis adde; et erit quattuor qui est radix census. Et census est sedecim. Omne autem quod fuerit maius censu uno aut minus, reduc ad censum unum.

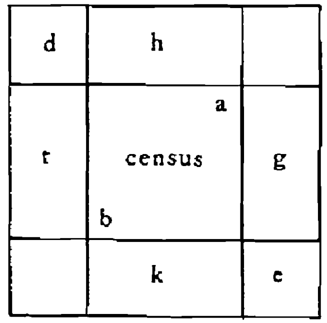
Hii ergo sunt sex modi, quos in huius nostri libri principio nominavimus. Et nos quidem iam explanavimus eos et diximus quod eorum tres modi sunt in quibus radices non mediantur. Quorum regulas et necessitates in precedentibus ostendimus. Illud vero quod ex mediatione radicum in tribus aliis capitulis est necessarium cum capitulis verificatis posuimus. Deinceps vero unicuique capitulo formam faciemus, per quam pervenitur ad causam mediationis.

< III. De demonstratione regularum >

Causa autem est ut hic. Census et decem radices equantur triginta novem dragmis. Fit ergo illi superficies quadrata ignotorum laterum, que est census quem et eius radices scire volumus. Que sit superficies *ab*. Unumquodque autem laterum ipsius est radix eius. Et unumquodque latus eius cum in aliquem numerum multiplicatur, tunc numerus qui inde aggregatur est numerus radicum quarum queque (P 111va) est sicut radix illius superficies. Postquam igitur dictum est quod cum censu sunt decem radices, accipiam quartam decem que est duo et semis. Et faciam unicuique quarte cum uno laterum superficiei superficiem. Fiunt ergo cum superficie prima que est superficies *ab* quattuor superficies equales cuiusque quarum longitudo est equalis radicis *ab* et latitudo est duo et semis. Que sunt superficies *g*, *h*, *t*, *k*. Radici igitur superficiei equalium laterum et etiam ignotorum deest quod ex angulis quattuor est

52 progreditur cum] probatur quod ex *F* 53-56 scias ... diminutione *om. ms. Vat. lat. 5733* 55 impossibilis] destructa falsa vel libera opinabilis *F*: destracta id est falsa vel inoppinabilis *M* 56 radicum] radicis *F* 72 formam] figuras *F*

III 5 aliquem] aliquo *L* 10 *post* prima *add.* postea *F* 11 quarum *om. F* *post* longitudo *add.* 4 superficium *F* 12 k] v *F* *post* superficiei *add.* ab *F* 13 et etiam] est *L* 13-14 quod ... deest *om. F*



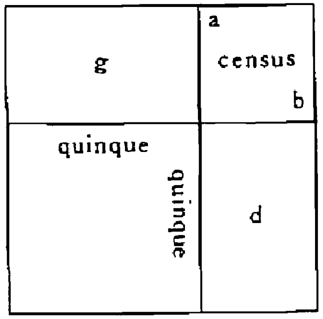
diminutum, scilicet unicuique angulorum deest multiplicatio duorum et semis in duo et semis. Quod igitur ex numeris necessarium est ad hoc ut superficiei quadratura compleatur, est multiplicatio duorum et semis in se quater. Et aggregatur ex summa illius totius viginti quinque. Iam autem scivimus quod prima superficies que est superficies census et quattuor superficies que ipsam circumdant, que sunt decem radices, sunt ex numeris triginta novem. Cum ergo addiderimus ei viginti quinque, qui sunt ex quattuor quadratis que sunt super angulos superficiei *ab*, complebitur quadratura maioris superficiei que est superficies *de*. Nos autem iam novimus quod totum illud est sexaginta quattuor. Unum igitur laterum eius est ipsius radix que est octo. Minuam itaque quod est equale quarte decem bis ab extremitatibus duabus lateris superficiei maioris que est superficies *de*. Et remanebit latus eius tres. Qui est equalis lateri superficiei prime, que est *ab*, et est radix illius census. Non autem mediamus radices decem et multiplicamus eas in se et addimus eas numero qui est triginta novem, nisi ut compleatur nobis figure maioris quadratura cum eo quod deest quattuor angulis. Cum enim cuiusque numeri quarta in se multiplicatur et deinde quod inde provenit in quattuor, erit quod proveniet multiplicationi medietati eius in se equale. Sufficit igitur nobis multiplicatio *medietatis* radicum in se, loco multiplicandi quartam in se quater.

Est eius preterea forma altera ad hoc idem perducens: que est superficies *ab* que est census. Volumus autem ut addamus ei equale decem radicibus eius. Mediabimus igitur decem et erunt quinque. Et faciemus eas duas superficies

15 ad hoc] adhuc *L* 18 que²] in *C* 20 ei] eis *CF* 23 minuam] minuas *L* 26 non] nos *L* 28 nobis *om. F* *post* figure *add.* quadrature *C* 29 deinde] deinceps *L* 29-30 quod inde *om. F* 30 multiplicationi] multitudo *F* 31-32 sufficit ... quater] non igitur curamus de multiplicatione medietatis radicum in se postquam eorum quartam in se quater multiplicavimus *F* 34 ut] quod *F*

34 *census*: see Euclid, *Elements*, book 2, prop. 4.

super duas partes *ab*, que sint due superficies *g* et *d* quarum cuiusque longitudo sit equalis lateri superficiei *ab*, et latitudo eius sit quinque que est medietas decem. Remanebit ergo nobis super superficiem *ab* quadratum quod fit ex quinque in quinque, qui est medietas decem radicum. Quas addidimus super

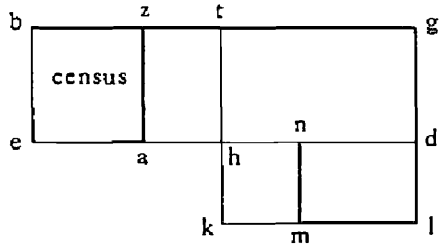


duas partes superficiei prime. Scimus autem quod superficies prima est census, et quod due superficies que sunt super duas ipsius partes sunt decem radices eius. Et hoc totum est triginta novem. Ad hoc igitur (P 111vb) ut maioris superficiei quadratum compleatur erit totum illud quod aggregatur sexaginta quattuor. Accipe ergo radicem eius que est unum laterum superficiei maioris: quod est octo. Cum ergo minuerimus ex ea equale ei quod super ipsam addidimus quod est quinque, remanebit tres. Qui est latus superficiei *ab* que est census. Ipse namque est radix eius, et census est novem.

Census autem et viginti unum equantur decem radicibus. Ponam itaque censum superficiem quadratam ignotorum laterum que sit superficies *ab*. Deinde adiungam ei superficiem equidistantium laterum cuius latitudo sit equalis uni lateri superficiei *ab*, quod sit latus *gd*. Et superficies sit *ga*. Et ponam ipsam esse viginti unum. Fit ergo longitudo duarum superficierum simul latus *ed*. Nos autem iam novimus quod longitudo eius est decem ex numeris. Omnis namque superficiei quadrate equalium laterum et angulorum, si unum latus multiplicatur in unum, est radix illius superficiei. Et si in duo, est due radices eius. Postquam igitur iam dictum est quod census et viginti una dragma equantur decem radicibus. Et scimus quod longitudo lateris *ed* est decem, quoniam latus *be* est radix census. Ergo dividam latus *ed* in duo media super

40 *post* prima *add.* scilicet ab *F* *variationes figurarum in codd. omisi* 42 ad hoc] adhuc *L* 43 quadratum] quadratura *L* 44 *post* unum *add.* (*et del. P*) quattuor *LP* 52 fit *om. L* simul *om. F* 55 *post* superficiei *add.* una *F* 58 *post* super *add.* ipsum *F*

49 *ab*: see Euclid, *Elements*, book 2, prop. 5 and p. 215 above.

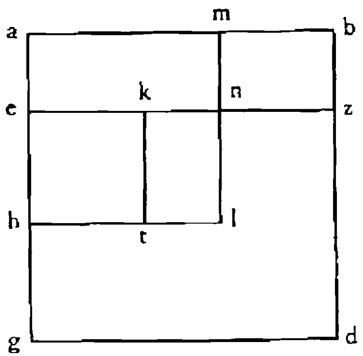


punctum *h*, et erigam super ipsum lineam *ht*. Manifestum est itaque quod *hd* est equalis *he*. Sed iam fuit nobis manifestum quod linea *ht* est equalis *be*. Addam itaque linee *ht* quod sit equale superfluo *dh* super *ht*, ut quadretur superficies, quod sit linea *hk*. Fit ergo *tk* equalis *tg*, quoniam *dh* fuit equalis *tg*; et provenit superficies quadrata que est superficies *lt*. Et ipsa est quod aggregatur ex multiplicatione medietatis radicum in se, que est quinque in quinque. Et illud est viginti quinque. Superficies vero *ag* fuit iam viginti unum qui iam fuit adjunctum ad censum. Post hoc faciamus super *hk* superficiem quadratam equalium laterum et angulorum, que sit superficies *mh*. Et iam scivimus quod *ht* est equalis *eb*. Sed *eb* est equalis *ae*. Ergo *ht* est equalis *ae*. Sed *tk* iam fuit equalis *he*. Ergo *ha* reliqua est equalis relique *hk*. Sed *hk* est equalis *mn*. Ergo *mn* est equalis *ha*. Sed *tk* iam fuit equalis *kl*, et *hk* est equalis *mk*. Ergo *ml* reliqua est equalis *ht* relique. Ergo superficies *ln* est equalis superficiei *ta*. Iam autem novimus quod superficies *lt* est viginti quinque. Nobis itaque patet quod superficies *gh* addita sibi superficiei *ln* est equalis superficiei *ga* que est viginti unum. Postquam ergo minuerimus ex superficie *lt* superficiem *gh* et superficiem *nl*, que sunt viginti unum, remanebit nobis superficies parva que est superficies *nk*. (P 112ra) Et ipsa est superfluum quod est inter viginti unum et viginti quinque. Et ipsa est quattuor cuius radix est *hk*. Sed ipsa est equalis *ha* et illud est duo. Sed *he* est medietas radicum, que est quinque. Cum ergo minuerimus ex ea *ha* que est duo, remanebit tres qui est linea *ae* que est radix census. Et census est novem. Et illud est quod demonstrare voluimus.

Dictum est autem 'Tres radices et quattuor dragme equantur censui.' Ponam ergo censum superficiem quadratam ignotorum laterum sed equalium et

59 hd] hoc *F* 60 sed] sic *L* (*et saepe infra*) 63 superficies[1] ... est[1] *om. C* 68-69 sed[2] ... he *om. F* 70 tk] ka *L* 74 *post* gh *add.* que est equalis superficiei gh et sit gh *F* 77 ha] hn *F* 78 *post* duo *add. F*: Sed he est me. Sed ipsa hk est equalis ha et sic est duo qui est radix quattuor in quo superat superficies lt superficiem ga. 80 et census *om. C* 82 sed equalium et] censum sed *F*

equalium angulorum, que sit superficies *ad*. Tota igitur hec superficies congregat tres radices et quattuor quos tibi nominavi. Omnis autem quadrate superficiei unum latus in unum multiplicatum est radix eius. Ex superficie igitur *ad* secabo superficiem *ed*, et ponam unum latus eius quod est *eg* tres qui est numerus radicum. Ipsum vero est equale *zd*. Nobis itaque patet quod superficies *eb* est quattuor qui radicibus est additus. Dividam ergo latus *eg* quod



est tres radices in duo media super punctum *h*. Deinde faciam ex eo superficiem quadratam que sit superficies *et*. Et ipsa est quod fit ex multiplicatione medietatis radicum, que est unum et semis in se, et est duo et quarta. Post hoc addam linee *ht* quod fit equale *ae* que sit linea *tl*. Fit ergo linea *hl* equalis *ah*, et provenit superficies quadrata que est superficies *hm*. Iam autem manifestum fuit nobis quod linea *ag* est equalis *ez*, et *ah* est equalis *en*. Remanet ergo *gh* equalis *nz*. Sed *gh* est equalis *kt*. Ergo *kt* est equalis *nz*. Sed *mn* est equalis *tl*. Superficies igitur *mz* fit equalis superficiei *kl*. Iam autem scivimus quod superficies *az* est quattuor qui est additus tribus radicibus. Fiunt ergo superficies *an* et superficies *kl* simul equales (P 112rb) superficiei *az* que est quattuor. Manifestum est igitur quod superficies *hm* est medietas radicum que est unum et semis in se, quod est duo et quarta, et quattuor additi qui sunt superficies *an* et superficies *kl*. Quod vero ex eo aggregatur est sex et quarta, cuius radix est duo et semis. Que est latus *ha*. Iam autem remansit nobis ex latere quadrati primi, quod est superficies *ad* que est totus census, medietas radicum que est unum et semis. Et est linea *gh*. Cum addiderimus super lineam *ah*, que est radix superficiei *hm* quod est duo et semis, lineam *hg* que est medietas radicum trium

84 congregat] aggregat *F* *post* nominavi *add.* in questione *F* 85 superficiei] superficie *L* 91 et quarta *om. F* 93 manifestum] illud *F* 98 equales] equal *L* 100-101 an et] autem *F* 105 est[1] *om. F* radicum *om. F* trium *om. C*

83 *ad*: see Euclid, *Elements*, book 2, prop. 6.

que est unum et semis, provenit illud totum quattuor. Quod est linea *ag*. Et ipsa est radix census qui est superficies *ad*. Et ipse est sedecim. Et illud est quod demonstrare voluimus.

Inveni autem omne quod fit ex computatione in algebra et almuchabala impossibile esse quin proveniat ad unum sex capitulorum que retuli tibi in principio huius libri.

<IV.> Capitulum multiplicationis

Nunc quidem refferam tibi qualiter res multiplicentur que sunt radices alie scilicet in alias cum fuerint singulares et cum numerus fuerit cum eis, aut fuerit exceptus ex eis numerus, aut ipse fuerint excepte ex numero, et qualiter alie aliis aggregentur, et qualiter alie ex aliis minuantur. Scias itaque impossibile esse quin unus omnium duorum numerorum, quorum unus in alterum multiplicatur, duplicetur secundum quantitatem unitatum que est in altero. Si ergo fuerit articulus et cum eo fuerint unitates aut fuerint unitates excepte ex eo, impossibile erit quin eius multiplicatio quater fiat; videlicet, articuli in articulum et unitatum in unitates, et unitatum in articulum et articuli in unitates. Quod si omnes unitates que sunt cum articulo fuerint addite aut diminute omnes, tunc quarta multiplicatio erit addita. Sin autem une earum fuerint addite et alie diminute, tunc quarta multiplicatio minuetur. Quod est sicut decem et unum in decem et duo. Ex multiplicatione igitur decem in decem fiunt centum. Et ex multiplicatione unius in decem fiunt decem addita. Et ex multiplicatione duorum in decem fiunt viginti addita. Et ex multiplicatione duorum in unum fiunt duo addita. Totum ergo illud est centum et triginta duo.

Et cum fuerint decem uno diminuto in decem uno diminuto, multiplicabis decem in decem et fient centum. Et unum diminutum in decem *et fient* decem diminuta. Et unum diminutum iterum in decem et fient decem diminuta. Unum quoque diminutum multiplicabis in unum diminutum, et fiet unum additum. Erit ergo totum illud octoginta unum.

Quod si fuerint decem et duo in decem uno diminuto, multiplicabis decem in decem et fient centum. Et unum diminutum in decem et erunt decem diminuta.

IV 1 *om. M* multiplicationis] de multiplicatione *F* 2 refferam] referatur *F* 3 scilicet] sunt *L* 4 ex[2]] in *C* 6 omnium] et medium *M* 7 *post* duplicetur *add.* vel multiplicetur *F* 9-11 videlicet ... unitates[1]] videlicet articuli in articulum et unitatum in unitates *F* 19 et[3] fient[2] *corr. ex* et fuit *P* et fient decem *om. F* 22 octoginta unum *om. F* 24 et fient centum *om. C*

IV 11 *unitates*[1]: the explanation follows from Euclid, *Elements*, book 2, prop. 1.

Et duo addita in decem et erunt viginti addita. Quod erit centum et de(P 112va)cem. Et duo addita in unum diminutum et erunt duo diminuta. Totum ergo illud erit centum et octo. Hoc autem non ostendi tibi nisi ut per ipsum perducaris ad multiplicationem rerum aliarum scilicet in alias, quin cum eis fuerit numerus aut cum ipse excipiuntur ex numero aut cum numerus excipitur ex eis.

Cumque tibi dictum fuerit: 'Decem dragme re diminuta – est enim rei significatio radix – multiplicate in decem', multiplicabis decem in decem et fient centum, et rem diminutam in decem et erunt decem res diminute. Dico igitur quod sunt centum, decem rebus diminutis. Si autem dixerit aliquis: 'Decem et res in decem', multiplica decem in decem et erunt centum, et rem additam in decem et erunt decem res addite. Erit ergo totum centum et decem res.

Quod si dixerit: 'Decem et res in decem et rem', dic: 'Decem in decem faciunt centum. Et res addita in decem facit decem res additas. Et res addita in decem facit etiam decem res additas. Et res addita in rem additam facit censum additum. Erit ergo totum centum et viginti res et census additus.' Quod si quis dixerit: 'Decem re diminuta in decem re diminuta', dices: 'Decem in decem fiunt centum. Et res diminuta in decem fit decem res diminute. Et res diminuta in decem fit decem res diminute. Et res diminuta in rem diminutam fit census additus. Est ergo illud centum et census additus diminutis viginti rebus.'

Et similiter si dixerit: 'Dragma minus sexta in dragma minus sexta', erit illud quinque sexte multiplicate in se, quod est viginti quinque partes triginta sex partium unius dragme. Regula vero eius est ut multiplices dragmam in dragmam et erit dragma, et sextam dragme diminutam in dragmam et erit sexta dragme diminuta. Et sextam diminutam in dragmam, et erit sexta diminuta. Fit ergo illud tertia dragme diminuta. Et sextam diminutam in sextam diminutam et erit sexta sexte addita. Totum igitur illud erit due tertie et sexta sexte.

Si vero aliquis dixerit: 'Decem re diminuta in decem et rem', dices: 'Decem in decem centum fiunt. Et res diminuta in decem fit decem res diminute. Et res in decem fit decem res addite. Et res diminuta in rem fit census diminutus. Est ergo illud centum dragme censu diminuto.' Si autem dixerit: 'Decem re diminuta in rem', dices: 'Decem in rem fiunt decem res. Et res diminuta in rem fit census diminutus. Sunt ergo decem res censu diminuto.' Et si dixerit: 'Decem et res in rem decem diminutis', dices: 'Res in decem fit decem res, et res in rem fit census. Et decem diminuta in decem fiunt centum dragme diminute. Et decem diminuta in re fiunt decem res diminute.' Dico igitur quod est census centum diminutis, postquam cum eo oppositum fuerit. Quod ideo est quoniam prohiciemus (P 112vb) decem res diminutas cum decem rebus additis, et remanebit census centum dragmis diminutis. Si autem dixerit quis: 'Decem dragme et medietas rei in medietatem dragme quinque rebus diminutis', dices: 'Medietas dragme in decem dragmas facit dragmas quinque. Et medietas dragme in medietatem rei facit quartam rei addite. Et quinque res diminute in decem dragmas fiunt quinquaginta res diminute. Et quinque res diminute in medietatem rei fiunt duo census et semis diminuti. Est ergo illud quinque dragme diminutis duobus censibus et semis, et diminutis quadraginta novem radicibus et tribus quartis radicis.'

Quod si aliquis dixerit tibi: 'Decem et res in rem diminutis decem', et est quasi dicat: 'Res et decem in rem decem diminutis', dic ergo: 'Res in rem facit censum. Et decem in rem fiunt decem res addite. Et decem diminuta in rem fiunt decem res diminute. Pretermittantur itaque addita cum diminutis, et remanebit census. Et decem diminuta in decem fiunt centum diminutum ex censu. Totum ergo illud est census diminutis centum dragmis.' Et omne quod est ex multiplicatione additi et diminuti, sicut res diminute in additam rem, in postrema multiplicatione semper minuitur.

< V. > Capitulum aggregationis et diminutionis

Radix ducentorum diminutis decem adiuncta ad viginti diminuta radice ducentorum est decem equaliter. Et radix ducentorum exceptis decem diminuta ex viginti excepta radice ducentorum est triginta diminutis duobus radicibus ducentorum. Et due radices ducentorum sunt radix octingentorum. Sed centum et census diminutis viginti radicibus, ad quem adiuncta sunt quinquaginta et decem radices diminutis duobus censibus, sunt centum et quinquaginta diminutis censu et decem radicibus. Ego vero illius causam in forma ostendam, si deus voluerit.

Scias itaque quod cum quamlibet census radicem notam sive surdam duplicare volueris, cuius duplicationis significatio est ut multiplices eam in duo, oportet ut multiplices duo in duo et deinde quod inde pervenerit in censum. Radix igitur eius quod aggregatur est duplum radicis illius census. Et cum

25 quod] abstrahas ergo 10 diminutas a 20 additis et *F* 26 duo^{2} *om*. *F* 31 *post* diminuta *add*. multiplica per 10 *FM* 32 in^{1}] per *FM* multiplicate in decem *om*. *M* 36 erit ... res *om*. *M* decem^{3}] 100 *F* 37 dic] duc *FM* 38-39 et^{2} ... additas *om*. *L* 40 *post* res *add*. radice *F* 42-43 et^{1} ... diminute *om*. *L* 45 *post* sexta^{2} *add*. *et del*. ei (?) *P* 46 quinque sexte] 2/6 *M* sexte] sex *C* 48 dragme diminutam] dragme minutam diminutam *C* 49 et^{2}] res *L* 50 in sextam diminutam *om*. *C* 55-57 si ... diminuto *om*. *M*

40 *additus*: see Euclid, *Elements*, book 2, prop. 4.

59-60 et^{2} ... diminute *om*. *L* 62 diminutas cum] de *M* 74 *post* res *add*. addite *C* 77 sicut] sunt *L*

V 1 *om*. *FM*

volueris triplum eius, multiplicabis tres in tres et postea quod inde provenerit in censum. Erit ergo radix eius quod aggregatur triplum radicis census primi. Et similiter quod additur ex duplicationibus aut minuitur erit secundum hoc exemplum.

Scias ergo ipsum quod si radicis census medietatem accipere volueris, oportet ut multiplices medietatem in medietatem, deinde quod provenerit in censum. Erit ergo radix eius quod aggregatur medietas radicis census. Et similiter si volueris tertiam aut quartam eius aut minus aut plus, usquequo possibile est consequi, secundum diminutionem et duplicationem. Verbi gratia: si enim volueris ut duplices radicem novem, multiplica duo in duo, postea (P 113ra) in novem et aggregatur triginta sex, cuius radix est sex. Qui est duplum radicis novem. Quod si ipsam volueris triplicare, multiplica tres in tres, postea in novem, et erunt octoginta unum, cuius radix est novem. Qui est radix novem triplicata. Sin autem radicis novem medietatem accipere volueris, multiplicabis medietatem in medietatem et proveniet quarta. Quam postea multiplicabis in novem. Et erunt duo et quarta cuius radix est unus et semis. Qui est medietas radicis novem.

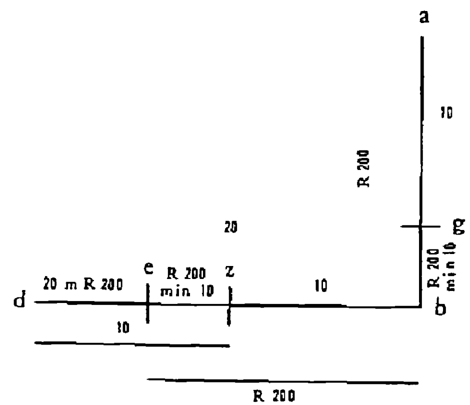
Et similiter quod additur aut minuitur ex noto et surdo erit. Et hic est eius modus. Quod si volueris dividere radicem novem per radicem quattuor, divides novem per quattuor et erunt duo et quarta. Cuius radix est id quod provenit uni quod est unus et semis. Quod si radicem quattuor per radicem novem volueris dividere, divide quattuor per novem et erunt quattuor none. Cuius radix est id quod provenit uni que est due tertie unius. Sin vero duas radices novem per radicem quattuor dividere volueris et absque hoc aliorum censuum, dupla ergo radicem novem secundum quod te feci noscere in opere multiplicium. Et quod aggregatur, divide per quattuor aut per quod volueris. Et quod ex censibus fuerit minus aut maius, secundum hoc exemplum operaberis per ipsum, si deus voluerit.

Quod si radicem novem in radicem quattuor multiplicare volueris, multiplica novem in quattuor et erunt triginta sex. Accipe igitur radicem eius que est sex. Ipse namque est radix novem in radicem quattuor. Et similiter si velles multiplicare radicem quinque in radicem decem, multiplicares quinque in decem et acciperes radicem eius. Et quod inde aggregaretur esset radix quinque in radicem decem. Quod si volueris multiplicare radicem tertie in radicem medietatis, multiplica tertiam in medietatem, et erit sexta. Radix ergo sexte est radix tertie in medietatem.

24 et aggregatur] est aggrega *L*   31 et[2]] ex *F*   33 erunt *om. L*   *post* uni *add.* et medietati radicis 4 *F*   37 dupla] multiplica *FM*   39 divide] deinde *L*   43 sex[1]] 9 *M*   43-44 erunt ... quattuor *om. F*   45 *post* multiplicare *add. et del.* volueris *P*

Sin autem duas radices novem in tres radices quattuor multiplicare volueris, producas duas radices novem secundum quod tibi retuli donec scias cuius census sit. Et similiter facias de tribus radicibus quattuor, donec scias cuius census sit. Deinde multiplica unum duorum censuum in alterum et accipe radicem eius quod aggregatur. Ipsa namque est due radices novem in tres radices quattuor. Et similiter de eo quod ex radicibus additur aut minuitur secundum hoc exemplum facias.

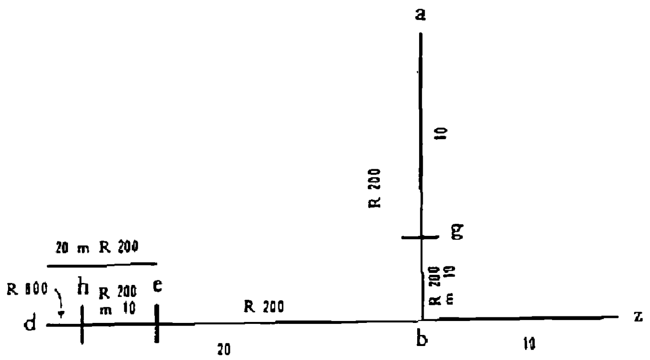
Cause autem radicis ducentorum diminutis decem, adiuncte ad viginti diminuta radice ducentorum, forma est linea *ab*. Ipsa namque est radix ducentorum. Ab *a* ergo ad punctum *g* est decem. Et residuum radicis ducentorum est residuum linee *ab* quod est linea *gb*. Deinde protraham a puncto *b* ad punctum *d* lineam que sit linea viginti. Ipsa namque est dupla linee *ag* que est decem. A puncto igitur *b* usque ad punctum *e* quod *e* sit equale linee *ab* que est radix ducentorum. Et residuum de viginti sit a puncto *e* usque ad punctum *d*. Et quia volumus aggregare quod remanet ex radice ducentorum



post proiectionem decem quod est linea *gb*, ad lineam *ed* que est viginti diminuta radice ducentorum. Et iam fuit nobis manifestum quod line(P 113rb)a *ab* que est radix ducentorum est equalis linee *be*, et quod linea *ag* que est decem est equalis linee *bz*, et residuum linee *ab* que est linea *gb* est equale residuo linee *be* quod est *ze*. Et addidimus super lineam *ed* lineam *ze*. Ergo manifestum est nobis quod iam minuitur ex linea *bd*, que est viginti, equale linee *ga* que est decem que est linea *bz*, et remanet nobis linea *zd* que est decem. Et illud est quod demonstrare voluimus.

51 duas radices *om. F*   58-59 forma ... ducentorum *om. C*   60 ducentorum] incentorum *L*   63 est *om. L*   *post* ducentorum *add.* et a puncto b usque ad punctum z sit 10 *F*   64 *post* d *add. F*: scilicet subiecta radice ducentorum que est linea bc, que postea equatur linea ab   70 nobis ... est[1] *om. F*

Causa vero radicis ducentorum exceptis decem diminute ex viginti excepta radice ducentorum est alia cuius forma est linea *ab* que est radix ducentorum. Sed ab *a* usque ad punctum *g* sit decem, qui est notus. Protraham autem a puncto *b* lineam usque ad punctum *d*, quam ponam viginti. Et ponam ut quod est a *b* usque ad punctum *e* sit equale radici ducentorum, que est equalis linee *ab*. Nobis vero iam fuit manifestum quod linea *gb* est id quod remanet ex radice



ducentorum post proiectionem decem, et linea *ed* est id quod remanet ex viginti post reiectionem radicis ducentorum. Volumus itaque ut linea *gb* minuatur ex linea *ed*. Protraham ergo a puncto *b* lineam ad punctum *z* que sit equalis linee *ag* que est decem. Fit ergo linea *zd* equalis linee *zb* et linee *bd*. Sed iam fuit nobis manifestum totum illud fore triginta. Secabo itaque ex linea *ed* quod sit equale linee *gb* quod est linea *he*. Patet igitur nobis quod linea *hd* est id quod remanet ex tota linea *zd* que est triginta. Ostensum vero est quod linea *be* est radix ducentorum, et linea *zb* et *bg* est etiam radix ducentorum. Et quia linea *eh* est equalis linee *gb*, ergo manifestum est quod illud quod minuitur ex linea *zd*, que est triginta, est due radices ducentorum. Et due radices ducentorum sunt radix octingentorum. Et illud est quod demonstrare voluimus.

Centum vero atque censui exceptis viginti radicibus, quibus coniunguntur quinquaginta et decem radices exceptis duobus censibus, non convenienter

75 ad punctum *om. F* 76-77 lineam ... b *om. F* 78 *in figuris codicum omnes descriptiones praeter litteras ad puncta omissae sunt* 81 b] db *L* 86 et bg *om. F* 88 que] quod *L* est[2]] et *L* 90 vero] ergo *L* censui] census *L* 91 convenienter] convenienti *L*

74 *ducentorum*[2]: although the explanation in the text is correct, the figure is faulty in one particular: *dh* does not equal R800 but rather 30 – R800. It is *hz* which equals R800. (*Note*: the figure here was drawn to scale, and the arrow was added to indicate where R800 is found in the figure in P).

subicitur forma ex tribus generibus divisis, scilicet censibus et radicibus et numero, neque cum eis quod eis equetur ut formentur. Nos tamen fecimus eis formam sed non sensibilem. Eorum vero necessitas verbis manifesta est. Quod est quare iam scivimus quod apud te sunt centum et census exceptis viginti radicibus. Postquam ergo addidisti eis quinquaginta et decem radices, facta sunt centum et quinquaginta et census exceptis decem radicibus. Hec namque decem radices addite restaurant viginti radicum diminutarum decem radices. Remanent ergo centum et quinquaginta et census, exceptis decem radicibus. Sed cum centum fuit iam census. Postquam ergo minueris duos census exceptos de quinquaginta, preteribit census cum censu, et remanebit tibi census. Fiet ergo centum et quinquaginta excepto censu, et exceptis decem radicibus. Et illud est quod demonstrare voluimus.

< VI. > Capitulum questionum

Iam processerunt ante capitula numerationis et eorum modos sex (P 113va) questiones quas posui exempla sex capitulis precedentibus in principio huius libri de quibus tibi dixi, quoniam impossibile est quin computatio algebre et almuchaba eveniat tibi ad aliquod capitulum eorum. Postea secutus sum illud ex questionibus cum eo quod intellectui propinquius fuit, per quod difficultas alleviabitur. Et significatio facilior fiet, si deus voluerit.

Questio earum prima est sicut si diceres: 'Divide decem in duas partes, et multiplica unam duarum partium in alteram; deinde multiplica unam earum in se. Et sit multiplicatio eius in se equalis multiplicationi uni duarum sectionum in alteram quater.' Eius vero regula est ut ponas unam duarum sectionum rem, et alteram sectionem ponas decem excepta re. Multiplica igitur rem in decem excepta re, et erunt decem res excepto censu. Deinde multiplica hoc totum in quattuor, que dixisti. Erit ergo quod proveniet quadruplum multiplicationis unius duarum sectionum in alteram. Erunt itaque quadraginta res exceptis quattuor censibus. Postea multiplica rem in rem que est una duarum sectionum in se; et erit census qui est equalis quadraginta rebus exceptis quattuor censibus. Deinde restaurabis quadraginta per quattuor census. Post hoc addes census

92 ex *om. L* 93 equetur] equantur *M* equetur ut formentur] sequitur ut formentur alie *F* 94 sed] suam *L* 95 quare] quod *L* 97 hec] hac *L* 101 census[1] *om. F* 102-103 et[3] ... voluimus *om. FM*

VI 1 *om. FM* 2 numerationis *om. FM* 11 *post* duarum *add. et del.* regularum *P* 13 re *om. C* multiplica] multiplicabis *L* 16 *post* rem[1] *add. F*: vocat hic aliam partem sectionis, scilicet maiorem quem ponit radicem multiplicando in se, et est ergo rem. 18 per *om. C* *post* census[1] *add. ms. Vat. lat. 5733*: Restaurare quadraginta radices per quatuor census est quatuor census diminutos a radicibus delere et illos quatuor numeros uno censu addere, nam si equalibus equalia addantur tota quoque fient equalia. Numerum enim dico quadraginta res exceptis quatuor censibus equantur uni censui. Dico quadraginta rebus deficere quatuor census censui. Et erit quod quadraginta res erunt equales quinque censibus. Ergo unus census erit octo radices qui est sexaginta quattuor. Radix ergo sexaginta quattuor est una duarum sectionum multiplicata in se. Et residuum ex decem est duo, qui est sectio altera. Iam ergo perduxi hanc questionem ad unum sex capitulorum, quod est quod census equatur radicibus.

Questio secunda: 'Divide decem in duas partes et multiplica decem in se. Et sit quod aggregatur ex multiplicatione decem in se equale uni duarum sectionum multiplicate in se bis et septem nonis vicis unius.' Computationis vero huius regula est ut ponas unam duarum sectionum rem. Multiplica igitur eam in se, et fiet census, deinde in duo et septem nonas. Erunt ergo duo census et septem none census unius. Deinde multiplica decem in se, et erunt centum. Est ergo ut centum sit equale duobus censibus et septem nonis census unius. Reduc ergo totum illud ad censum unicum, qui est novem partes viginti quinque, quod est quinta et quattuor quinte quinte unius. Accipe igitur quintam centum et quattuor quintas quinte ipsius, que sunt triginta sex. Et ipse equantur censui cuius radix est sex, qui est una duarum sectionum. Iam ergo produximus hanc questionem ad unum sex capitulorum, quod est quod census equatur numero.

Questio tertia: 'Divide decem in duas sectiones et divide unam duarum partium per alteram, et provenient quattuor.' Cuius regula est ut ponas unam duarum sectionum rem et alteram decem excepta re. Deinde dividas decem excepta re per rem, ut proveniat quattuor. Iam autem scivisti quod cum multiplicaveris quod provenit ex divisione in idem per quod divisum (P 113vb) fuit, redibit census tuus quem divisisti. Sed proveniens ex divisione in hac questione fuit quattuor et id, per quod divisum fuit, fuit res. Multiplica igitur quattuor in rem, et erunt quattuor res. Ergo quattuor res equantur censui quem divisisti, qui est decem excepta re. Restaura itaque decem per rem, et adde ipsam quattuor. Erit ergo quod decem equatur quinque rebus. Ergo res est duo. Iam ergo perduxi hanc questionem ad unum sex capitulorum, quod est quod radices equantur numero.

Questio quarta: 'Multiplica tertiam census et dragmam in quartam eius et dragmam, et sit quod provenit viginti.' Cuius regula est ut tu multiplices tertiam

ut equentur uni censui et ut ad regulam reducamus. Compleo quadraginta res auferendo negationem quatuor censuum. Et sic in complemento quadraginta rerum sunt additi quatuor census. Totidemque operatione addere uni censui, et exeunt quadraginta res equalium quinque censibus. 24 questio secunda *om. FM* 29 et septem none census *om. F* 31 *post* partes *add.* de *F* 32 *post* quinque *add. F*: scilicet partibus que sicut dicunt est equalis 100 quod est novenarius 11 (?) constituitur ex denario quod est quinta pars 25 et quoniam qui est quartum quinte quinarii, id est, ex quo 24 quintis quinte, id est, 25 quatuor est 4/5 unius quinte. 33-34 centum ... sectionum *om. C* 37-48 questio ... numero *om. F* 47 perduxi] produxi *L* (*et alibi infra*) 49 questio quarta *om. M*

in quartam, et erit quod proveniet medietas sexte census. Et dragmam in dragmam, et erit dragma addita. Et tertiam rei in dragmam, et erit tertia radicis. Et quartam rei in dragmam, et erit quarta radicis. Erit ergo illud medietas sexte census et tertia rei et quarta rei et dragma, que equatur viginti dragmis. Prohice ergo dragmam unam ex viginti dragmis et remanent decem et novem dragme, que equantur medietati sexte census et tertie et quarte radicis. Reintegra ergo censum tuum. Eius vero reintegratio est ut multiplices totum quod habes in duodecim, et provenient tibi census et septem radices, que erunt equales ducentis et viginti octo. Media igitur radices et multiplica eas in se, que erunt duodecim et quarta. Et adde eas ducentis et viginti octo. Erit ergo illud ducenta et quadraginta et quarta. Deinde accipe radicem eius que est quindecim et semis. Ex qua minue medietatem radicum que est tres et semis. Remanet ergo duodecim qui est census. Iam ergo perduximus hanc questionem ad unum sex capitulorum, quod est quod census et radices equantur numero.

Questio quinta: 'Divide decem in duas partes, et multiplica unamquamque earum in se et aggrega eas. Et proveniat in quinquaginta octo.' Cuius regula est ut multiplices decem excepta re in se, et provenient centum et census exceptis viginti rebus. Deinde multiplica rem in se et erit census. Postea aggrega ea et erunt centum nota et duo census exceptis viginti rebus, que equantur quinquaginta octo. Restaura ergo centum et duos census per res que fuerunt diminute, et adde eas quinquaginta octo. Et dices: 'Centum et duo census equantur quinquaginta octo et viginti rebus.' Reduc ergo ea ad censum unum. Dices ergo: 'Quinquaginta et census equantur viginti novem et decem rebus.' Oppone ergo per ea. Quod est ut tu prohicias ex quinquaginta viginti novem. Remanet ergo viginti unum et census, que equantur decem rebus. Media ergo radices, et provenient quinque. Eas igitur in se multiplica, et erunt viginti quinque. Prohice itaque ex eis viginti unum, et remanebunt quattuor. Cuius radicem accipias que est duo. Minue ergo ipsam ex quinque rebus, que sunt medietas radicum; et remanet tres, qui est una duarum sectionum. Iam ergo perduximus hanc questionem (P 114ra) ad unum sex capitulorum, quod est: census et numerus equantur radicibus.

Questio sexta: 'Tertia census multiplicetur in quartam eius, et proveniat inde census. Et sit augmentum eius viginti quattuor.' Cuius regula est quoniam tu nosti quod cum tu multiplicas tertiam rei in quartam rei, provenit medietas sexte census que est equalis rei et viginti quattuor dragmis. Multiplica igitur

54 prohice] projici *L* (*et alibi infra*) 59 viginti *om. F* *post* media *add.* tuis *F* 61 quadraginta et quarta] viginti *F* 62-63 ex ... census] remanet ergo 12, supradictis 3 et semis, quod est medietas radicis quod est census *F* 65 questio quinta *om. FM* divide] siuidetur *F* 68 aggrega] aggregata *FM* 69 nota] tota *F* 77 ex *om. C* 82 questio sexta *om. CFM* 85 *post* sexte *add. et del.* casus *P*

medietatem sexte census in duodecim ut census reintegretur et fiat census perfectus. Et multiplica etiam rem et viginti quattuor in duodecim et provenient tibi ducenta et octoginta octo et duodecim radices, que sunt equales censui. Media igitur radices et multiplica eas in se. Quas adde ducentis et octoginta octo; et erunt omnia trecenta et viginti quattuor. Deinde accipe radicem eius que est decem et octo. Cui adde medietatem radicum, et fiet census viginti quattuor. Iam igitur perduximus hanc questionem ad unum sex capitulorum, quod est: numerus et radices equantur censui.

< VII. Questiones varie >

< 1 > Quod si aliquis interrogans quesierit et dixerit: 'Divisi decem in duas partes. Deinde multiplicavi unam earum in alteram et provenerunt viginti unum.' Tu ergo iam scivisti quod una duarum sectionum decem est res. Ipsam igitur in decem, re excepta, multiplica, et dicas: 'Decem excepta re in rem sunt decem res, censu diminuto, que equantur viginti uno.' Restaura igitur decem excepta re per censum, et adde censum viginti uno; et dic: 'Decem res equantur viginti uno et censui.' Radices ergo mediabis et erunt quinque. Quas in se multiplicabis et proveniet viginti quinque. Ex eo itaque prohice viginti unum, et remanet quattuor. Cuius accipe radicem que est duo, et minue eam ex medietate rerum. Remanet ergo tres qui est una duarum partium.

< 2 > Quod si dixerit: 'Divisi decem in duas partes et multiplicavi unamquamque earum in se. Et minui minus ex maiore et remanserunt quadraginta.' Erit eius regula ut multiplices decem excepta re in se et provenient centum et census, viginti rebus diminutis. Et multiplica rem in rem, et erit census. Ipsum ergo minue ex centum et censu exceptis viginti rebus. Remanet itaque centum exceptis viginti rebus que equantur quadraginta. Restaura ergo centum per viginti, et adde ipsum quadraginta. Habebis ergo quadraginta et viginti res que erunt equales centum. Oppone igitur per eas centum; prohice quadraginta ex centum. Remanent sexaginta que equantur viginti rebus. Ergo res equatur tribus, qui est una duarum partium.

< 3 > Si autem dixerit: 'Divisi decem in duas partes et multiplicavi unamquamque partem in se, et aggregavi eas. Et insuper addidi eis superfluum

91 decem et octo] 88 *F*

VII 8 mediabis] media *FM* 9 multiplicabis] multiplica *F* 16 et censu] et censu ex censu *F* 16-17 remanet ... rebus *om.* *L* 18 ipsum] eis *F* 19 oppone] appone *L* (*et alibi infra*) per *om.* *C* centum[1]] censui *L* 20 ex centum *om.* *M* equantur] erit equalis *F*

VI 87 *perfectus*: this is the only place in the text where a square is called a 'perfect square'.

quod fuit inter utrasque sectiones antequam in se multiplicarentur. Et provenit illud totum quinquaginta quattuor.' Regula itaque eius est ut multi(P 114rb)plices decem excepta re in se; et erit quod proveniet centum et census exceptis viginti rebus. Ex decem vero remansit res. Multiplica ergo ipsam in se, et erit quod proveniet census. Deinde aggrega ea, et erit illud quod proveniet centum et duo census exceptis viginti rebus. Adde igitur superfluum quod fuit inter eas aggregato, quod est decem exceptis duabus rebus. Totum ergo illud est centum et decem et duo census exceptis duabus rebus et exceptis viginti rebus, que equantur quinquaginta quattuor dragmis. Cum ergo restaurabis, dices: 'Centum et decem dragme et duo census equantur quinquaginta quattuor et viginti duabus rebus.' Reduc ergo ad censum suum. Et dic: 'Census et quinquaginta quinque equantur viginti septem dragmis et undecim rebus.' Prohice ergo viginti septem et remanebunt census et viginti octo que equantur undecim rebus. Media igitur res et erunt quinque et semis. Et multiplica eas in se, et erunt triginta et quarta. Ex eis igitur minue viginti octo. Et residui radicem sume, quod est duo et quarta. Est ergo unum et semis. Et minue eam ex medietate radicum et remanebunt quattuor, qui est una duarum partium.

< 4 > Quod si dixerit: 'Divisi decem in duas partes et divisi hanc per illam et illam per istam. Et provenerunt due dragme et sexta.' Huius autem regula est. Quoniam cum tu multiplicabis unamquamque partem in se et postea aggregabis eas, erit sicut cum una duarum partium multiplicatur in alteram. Et deinde quod provenit multiplicatur in id quod aggregatur ex divisione, quod est duo et sexta. Multiplica igitur decem excepta re in se, et erunt centum et census exceptis viginti rebus. Et multiplica rem in rem, et erit census. Aggrega ergo illud. Et habebis centum et duo census exceptis viginti rebus, que equantur rei multiplicate in decem minus re. Que est decem res excepto censu multiplicato in id quod provenit ex duabus divisionibus, quod est duo et sexta. Erit ergo illud viginti et una res et due tertie radicis exceptis duobus censibus et sexta, que equantur centum et duobus censibus exceptis viginti rebus. Restaura ergo illud, et adde duos census et sextam centum et duobus censibus exceptis viginti rebus. Et adde viginti res diminutas ex centum, viginti uni et duabus tertiis radicis. Habebis ergo centum et quattuor census et sextam census que equantur quadraginta uni rei et duabus tertiis rei. Reduc ergo illud ad censum unum. Tu autem iam scivisti quod unus census quattuor censuum et sexte est quinta et

29-30 eas aggregato] eis aggregatis *F* 31 viginti] 22 *M* viginti rebus que] 20 que scilicet omnia simul aggregata *F* 34 suum] unum *FM* 35 quinque *om.* *F* viginti septem] 28 *F* undecim] 10 *F* 36 que] qui *L* 45 in id quod aggregatur] et 10 quod multiplicatur *F* id] illud *C* 49 minus ... res *om.* *F* *post* censu *add.* residuum *M* 50 quod[2]] que *L* 51 due tertie] tertiam *F* 55 sextam census] sexta in censu *C* 56 *post* tertiis *add. et del.* rei *P* reduc] rebus *L* 57 iam] eam *L* sexte est quinta] 8a est 4a *F* 57-58 quinta et quinta quinte] quinta quinte *L* quinta quinte. Totius igitur quod habes accipe quintam et quintam quinte; et habebis censum et viginti quattuor dragmas que equantur decem radicibus. Media ergo radices et multiplica eas in se. Et erunt viginti quinque ex quibus minue viginti quattuor que sunt cum censu, et remanebit unum. Cuius assume radicem que est unus. Ipsam ergo minue ex medietate radicum que est quinque. Et remanet quattuor, qui est una duarum sec(P 114va)tionum. Et provenit ex hoc ut cum illud quod provenit ex divisione quarumlibet duarum rerum, quarum una per alteram dividitur, multiplicatur in id quod provenit ex divisione alterius per primum, erit semper quod proveniet unum.

< 5 > Sin vero dixerit: 'Divisi decem in duas partes et multiplicavi unam duarum partium in quinque et divisi quod aggregatum fuit per alteram. Deinde proieci medietatem eius quod provenit et addidi ipsam multiplicato in quinque. Et fuit quod aggregatum est quinquaginta dragme.' Erit huius regula ut ex decem accipias rem et multiplices eam in quinque. Erunt ergo quinque res divise per secundam que est decem excepta re, accepta eius medietate. Cum ergo acceperis medietatem quinque rerum que est duo et semis, erit illud quod vis dividere per decem excepta re. He ergo due res et semis divise per decem excepta re, equantur quinquaginta exceptis quinque rebus. Quoniam dixit: 'Adde ipsam uni duarum sectionum multiplicate in quinque', est ergo totum illud quinquaginta. Iam autem scivisti quod cum multiplicas quod provenit tibi ex divisione in id per quod dividitur, redit census tuus. Tuus autem census est due res et semis. Multiplica ergo decem excepta re in quinquaginta exceptis quinque rebus. Erit itaque quod proveniet quingenta et quinque census exceptis centum rebus, que equantur duabus rebus et semis. Reduc ergo illud ad censum unum. Erit ergo quod centum dragme et census exceptis viginti rebus equantur medietati rei. Restaura igitur centum et adde viginti res medietati rei. Habebis ergo centum dragmas et censum que equantur viginti rebus et medietati rei. Ergo media radices et multiplica eas in se, et minue ex eis centum, et accipe residui radicem, et minue eam ex medietate radicum que est decem et quarta. Et remanebit octo que est una duarum sectionum.

< 6 > Quod si aliquis dixerit tibi: 'Divisi decem in duas partes et multiplicavi unam duarum partium in se. Et fuit quod provenit equale alteri parti octuagies et semel.' Erit huius regula ut dicas: 'Decem excepta re in se fiunt centum et census exceptis viginti rebus, que equantur octoginta uni rei.' Restaura ergo centum, et adde viginti radices octoginta uni. Erit ergo quod

58 quintam[2]] quinta *L* 62 radicum] radicem *L* 63 duarum] earum *F* 64 duarum] duorum *L* 67 in duas partes *om. F* 74-75 he ... re *om. F* 90 decem] 4ta *F*

centum et census erunt equales centum radicibus et uni radici. Media igitur radices et erunt quinquaginta et semis. Multiplica eas in se et erunt bis mille *et quingente et quinquaginta et quarta. Ex eis itaque minue centum. Et remanebunt bis mille* et quadringente et quinquaginta et quarta. Accipe igitur eius radicem que est quadraginta novem et semis. Et minue eam ex medietate radicum que est quinquaginta et semis. Et remanebit unus qui est una duarum sectionum.

< 7 > Et si aliquis dixerit: 'Duo census sunt inter quos sunt due dragme quorum minorem per maiorem divisi, et (P 114vb) provenit ex divisione medietas.' Dic: *'Hic rem ponit pro censu.'* Ergo res et due dragme in medietatem, que est id quod provenit ex divisione, est medietas rei et dragma, que sunt equales rei. Prohice ergo medietatem rei cum medietate, et remanet dragma que est equalis medietati rei. Dupla ergo, et dic ergo quod res est due dragme et altera est quattuor.

< 8 > Quod si dixerit tibi: 'Divisi decem in duas partes. Deinde multiplicavi unam earum in alteram. Et post divisi quod aggregatum fuit ex multiplicatione per superfluum quod fuit inter duas sectiones antequam una in alteram multiplicaretur. Et provenerunt quinque et quarta.' Erit eius regula ut accipias ex decem rem, et remanebunt decem excepta re. Unum igitur multiplica in alterum et erunt decem radices excepto censu. Et hoc est quod provenit ex multiplicatione unius eorum in alterum. Deinde divide illud per superfluum, quod est inter ea, quod est decem exceptis duabus rebus. Provenit ergo quinque et quarta. Cum ergo multiplicaveris quinque et quartam in decem exceptis duabus rebus, proveniet inde census multiplicatus qui est decem res excepto censu. Multiplica ergo quinque et quartum in decem exceptis duabus rebus. Et erit quod proveniet quinquaginta due dragme < et semis > exceptis decem radicibus et semis, que equantur decem radicibus excepto censu. Restaura ergo quinquaginta duo et semis per decem radices et semis, et adde eas decem radicibus excepto censu. Deinde restaura eas per censum et adde censum quinquaginta duobus et semis. Et habebis viginti radices et semis que equantur

94 *post* mille *add. et del.* et quingente et quinquaginta quarta *P* 96 bis mille *om. F* 102-104 dic ... rei[1]] dragme. Pro minori censu pone rem. Pro maiori vero censu pone rem et duas dragmas. Quibus multiplicatis per mediam dragmam que provenit ex divisione minoris censi per maiorem et eveniunt media res et dragmam, id est, que equantur uni rei *M* 105 dupla ergo et *om. F* 119 decem *om. F*

118 *et semis*: from the margin and in a contemporary hand. The omission of these words from the text proper of P is clearly a scribal error, as the next sentence suggests.

quinquaginta duabus dragmis et semis et censui. Operaberis ergo per eas secundum quod posuimus in principio libri, si deus voluerit.

<9> Si quis vero tibi dixerit: 'Est census cuius quattuor radices multiplicate in quinque radices ipsius reddunt duplum census et augent super hoc triginta sex dragmas.' Huius regula est. Quoniam cum tu multiplicas quattuor radices in quinque radices, fiunt viginti census qui equantur duobus censibus et triginta sex dragmis. Prohice ergo ex viginti censibus duos census cum duobus censibus. Ergo remanent decem et octo census qui equantur triginta sex. Divide igitur triginta sex per decem et octo. Et proveniet duo qui est census.

<10> Quod si dixerit: 'Est census cuius tertia et tres dragme, si auferantur et postea multiplicetur quod remanet in se, redibit census.' Erit eius regula. Quoniam cum tu proieceris tertiam et tres dragmas, remanebunt eius due tertie exceptis tribus dragmis, que est radix. Multiplica igitur duas tertias rei, *id est census*, exceptis tribus dragmis in se. Due ergo tertie multiplicate in duas tertias fiunt quattuor none census. Et tres dragme diminute in duas tertias rei, due radices sunt. Et tres diminu(P 115ra)te in duas tertias faciunt duas radices, et tres in tres fiunt novem dragme. Sunt ergo quattuor none census et novem dragme exceptis quattuor radicibus que equantur radici. Adde ergo quattuor radices radici. Et erunt quinque radices que erunt equales quattuor nonis census et novem dragmis. Cum ergo vis ut multiplices quattuor nonas donec reintegres censum tuum, multiplica igitur omne quattuor in duo et quartam, et multiplica novem in duo et quartam. Et erunt viginti dragme et quarta. Et multiplica quinque radices in duo et quartam, et erunt undecim res et quarta. Facies ergo per ea sicut est illud quod retuli tibi de mediatione radicum, si deus voluerit.

<11> Et si dixerit: 'Dragma et semis fuit divisa per hominem et partem hominis, et evenit homini duplum eius quod accedit parti.' Erit eius regula ut dicas: 'Homo et pars est unum et res.' Est ergo quasi dicat: 'Dragma et semis dividitur per dragmam et rem, et proveniunt dragme due res.' Multiplica ergo duas res in dragmam et rem. Et provenient duo census et due res que equantur dragme et semis. Reduc ea ad censum unum. Quod est ut accipias ex

127 triginta] et *F* 128 in quinque radices *om. L* 129 triginta sex] 28 *F* 134 due tertie *om. F* 135 tribus] duobus *F* 135-136 id est census *om. C* 138 et¹ ... radices *om. F* 143 censum tuum] 9 radices *FM* omne *om. CFM* 144 et² ... quarta *om. M* 145 facies] fac *L*

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123 *operaberis*: a contemporary hand adds in the margin of P *in libro erat oppones* as an explanation of *operaberis*; the same addition is found in Q. 133 *census*: the *census* is the square of $(^2/_3\, x - 3)$ which equals *radix* in line 135. 145 *facies*: a contemporary hand adds in the margin of P the gloss *vel oppones*; the same gloss is found in Q.

unaquaque re ipsius medietatem. Et dicas: 'Census et res equantur tribus quartis dragme.' Oppone ergo per ea secundum quod ostendi tibi.

<12> Quod si dixerit tibi: 'Divisi dragmam per homines, et provenit eis res. Deinde addidi eis hominem. Et postea divisi dragmam per eos, et provenit eis minus quam ex divisione prima secundum quantitatem sexte dragme unius.' Erit eius consideratio ut multiplices homines primos in diminutum quod est inter eos. Deinde multiplices quod aggregatur per illud quod est inter homines primos et postremos. Proveniet ergo census tuus. Multiplica igitur numerum primorum hominum qui est res in sextam que est inter eos, et erit sexta radicis. Deinde multiplica illud in numerum hominum posteriorum, qui est res et unum. Erit ergo quod sexta census et sexta radicis divisa per dragmam equatur dragme. Ergo reintegra illud: multiplica ipsum in sex, et erit quod habebis census et radix. Et multiplica dragmam in sex, et erunt sex dragme. Census ergo et radix equantur sex dragmis. Media ergo radices et multiplica eas in se et adde eas super sex. Et accipe radicem eius quod aggregatur et minue ex ea medietatem radicis. Quod ergo remanet est numerus hominum primorum, qui sunt duo homines.

<VIII.> Capitulum conventionum negociatorum

Scias quod conventiones negociationis hominum omnes, que sunt de emptione et venditione et cambitione et conductione et ceteris rebus, sunt secundum duos modos, cum quattuor numeris quibus interrogator loquitur. Qui sunt pretium et appretiatum secundum positionem, et pretium et appretiatum secundum querentem. Numerus vero qui est appretiatum secundum positionem opponitur numero qui est pretium secundum querentem. Et numerus qui est pretium secundum positionem opponitur numero qui est appretiatum secundum querentem. Ho(P 115rb)rum vero quattuor numerorum tres semper manifesti et noti, et unus *est* ignotus. Qui est ille qui verbo loquentis notatur per quartum, et de quo interrogator querit. Regula ergo in hoc est ut consideres tres numeros manifestos. Impossibile est enim quin duo eorum sint quorum unusquisque suo compari est oppositus. Multiplica igitur unumquemque duorum numerorum apparentium oppositorum in alterum. Et quod proveniet, divide per alterum numerum cui numerus ignotus opponitur. Quod

156 deinde ... eis *om. F* 159 est] et *L* 160 ergo] ego *L*
VIII 1 *om. F* negociatorum *om. M* 2 omnes *om. L* 11 in hoc] hec *L*

VIII 1 The chapter is an application of 'The Rule of Three'; that is, given three of four terms in proportion, the fourth is easily found. See Euclid, *Elements*, book 7, prop. 19.

ergo proveniet, est numerus ignotus pro quo querens interrogat. Qui etiam est oppositus numero per quem dividitur.

Cuius exemplum secundum primum modum eorum est ut querens interroget et dicat: 'Decem cafficii sunt pro sex dragmis; quot ergo provenient tibi pro quattuor dragmis?' Sermo itaque eius, qui est decem cafficii, est numerus appretiati secundum positionem. Et eius sermo, qui est sex dragme, est numerus eius quod est pretium secundum positionem. Et ipsius sermo, quo dicitur quantum te contingit, est numerus ignotus appretiati secundum querentem. Et ipsius sermo, qui est per quattuor dragmas, est numerus qui est pretium secundum querentem. Numerus ergo appretiati qui est decem cafficii opponitur numero qui est pretium secundum querentem, quod est quattuor dragme. Multiplica ergo decem in quattuor, qui sunt oppositi et manifesti, et erunt quadraginta. Ipsum itaque per alium numerum manifestum divide, qui est pretium secundum positionem, quod est sex dragme. Erit ergo sex et due tertie qui est numerus ignotus. Qui est sermo dicentis quantum. Ipse namque est appretiatum secundum querentem, et opponitur sex qui est pretium secundum positionem.

Modus autem secundus est sermo dicentis: 'Decem sunt pro octo; quantum est pretium quattuor?' Aut forsitan dicitur: 'Quattuor eorum quanti pretii sunt.' Decem ergo est numerus appretiati secundum positionem. Et ipse opponitur numero qui est pretii ignoti, qui notatur per verbum illius 'quantum'. Et octo est numerus qui est pretium secundum positionem. Ipse namque opponitur numero manifesto qui est appretiati qui est quattuor. Multiplica ergo duorum numerorum manifestorum et oppositorum unum in alterum, scilicet quattuor in octo, et erunt triginta duo. Et divide quod proveniet per alium numerum manifestum, qui est appretiati, et est decem. Erit ergo quod perveniet tres et quinta, qui est numerus qui est appreciatum. Et ipse est oppositus decem per quem divisum fuit. Et similiter erunt omnes conventiones negociationis et earum regule.

Quod si aliquis querens interrogaverit et dixerit: 'Quemdam operarium conduxi in mense pro decem dragmis, qui sex diebus operatus est; quantum ergo contigit eum?' Tu autem iam scivisti quod sex dies sunt quinta mensis, et quod illud quod ipsum contingit ex dragmis est secundum quantitatem eius quod operatus est ex mense. Eius vero regula est quod mensis est triginta dies quod est appretiatum secundum positionem. Et sermo eius qui est decem est pretium secundum positionem. Eius vero sermo qui est sex dies est appretiatum secundum querentem. Et sermo eius quantum contigit est pretium secundum

19, 20 cafficii] kaficii *CF*: radicis *M* 25 est] et *L* cafficii] kaficii *CF* 26 pretium *om. F* 37 *post* numerus *add. et del.* quantum *P* 38-39 duorum ... oppositorum] duos numeros manifestos *F*: numeros manifestos *M* 46 decem] 6 *F* 51 sex] 30 *M*

querentem. Multiplica ergo pretium secundum positionem, quod est decem, in appretiatum secundum querentem, quod est ei oppositum et est sex. (P 115va) Et provenient sexaginta. Ipsum ergo divide per triginta qui est numerus manifestus qui est appretiatum secundum positionem. Erit ergo illud due dragme quod est pretium secundum querentem. Et similiter fiunt omnia quibus homines inter se conveniunt in negociatione, secundum cambium et mensurationem et ponderationem.

< Appendix >

Liber hic finitur. In alio tamen *libro* repperi hec interposita suprascriptis.

< 1 > *Iterata* quod si quis dixerit tibi: 'Divisi decem in duas partes et multiplicavi unam duarum sectionum in se. Et fuit quod provenit equale alteri octuagies et semel.' Erit eius regula ut dicas: 'Decem excepta re in se fiunt centum et census exceptis viginti rebus que equantur octoginta uni rei.' Restaura ergo centum et adde viginti radices octoginta uni et erunt centum et census, que erunt equales centum et uni radici. Radices igitur mediabis et erunt quinquaginta et semis. Multiplica ergo eas in se, et erunt bis mille et quingente et quinquaginta et quarta. Ex quibus minue centum, et remanebunt bis mille et quadringente et quinquaginta et quarta. Huius itaque accipe radicem. Que est quadraginta novem et semis. Quam minuas ex medietate radicum, que est quinquaginta et semis. Et remanebit unum, qui est una duarum sectionum.

< 2 > Si autem aliquis dixerit: 'Divisi decem in duas partes et multiplicavi unam duarum partium in decem et alteram in se, et fuerunt equales.' Erit eius regula ut multiplices rem in decem, et erunt decem radices. Deinde multiplica decem excepta re in se, et erunt centum et census exceptis viginti rebus. Que equantur decem radicibus. Oppone ergo per eas.

< 3 > Quod si dixerit: 'Due tertie quinte census, septime radicis ipsius sunt equales. Tunc tota radix equatur quattuor quintis census et duabus tertiis quinte ipsius, que est quattuordecim partes de quindecim.' Erit huius regula ut multiplices duas tertias quinte in *septem* ut radix compleatur. Due vero tertie quinte sunt due partes *de* quindecim. Multiplica igitur quindecim in se, et erunt ducenta et viginti quinque, et quattuordecim in se, et erunt centum et nonaginta sex. Minue igitur ex ducentis viginti quinque duas

55 sexaginta] 16 *F* 56 due *om. L*
Appendix 2 hic *om. C* libro *om. CL* 2-12 liber ... sectionum *om. M* 3 iterata *om. CFL* 4 sectionum] partium *F* 5 re *om. L* 14 eius] hec *L* 16 radicibus] rebus *F* 19 tunc *om. L* que] qua *L* 21 septem] se (septem *in marg.*) *P*

Appendix 1 The Appendix added by Gerard ('repperi') is not found in Robert of Chester's translation (see Karpinski, *Robert of Chester*, p. 124) nor in the translation ascribed to William of Lunis.

tertias quinte ipsius que est triginta, et erit pars de quindecim. Quam dividas per septimam diminutam ex centum nonaginta sex que est viginti octo. Et proveniet unum et quarta decima unius, que est media septima et est radix census.

< 4 > Si autem dixerit: 'Multiplicavi censum in quadruplum ipsius et provenerunt viginti.' Erit eius regula. Quoniam cum tu multiplicas ipsum in se, provenit quinque. Ipse namque est radix quinque.

< 5 > Quod si dixerit: 'Est census quem in sui tertiam multiplicavi, et provenit decem.' Erit eius consideratio. Quoniam cum tu multiplicas ipsum in se, provenit triginta. Dic ergo quod census est radix triginta.

< 6 > (P 115vb) Si dixerit: 'Est census quem in quadruplum ipsius multiplicavi, et provenit tertia census primi.' Erit eius regula. Quoniam si tu multiplicaveris ipsum in duodecuplum ipsius, proveniet quod erit equale censui. Quod est medietas sexte in tertiam.

< 7 > Quod si dixerit: 'Est census quem multiplicavi in radicem ipsius, et provenit triplum census primi.' Erit eius consideratio. Quoniam cum tu multiplicas radicem census in tertiam ipsius, provenit census. Dico igitur quod istius census tertia est radix eius. Et ipse est novem.

< 8 > Si vero dixerit: 'Est census cuius tres radices in ipsius quattuor radices multiplicavi, et provenit census et augmentum quadraginta quattuor.' Erit regula huius. Quoniam cum tu multiplicas quattuor radices in tres radices, fiunt duodecim census. Qui sunt equales censui et quadraginta quattuor dragmis. Ex duodecim igitur censibus prohice censum unum. Remanent ergo undecim census equales quadraginta quattuor. Divide itaque quadraginta quattuor per undecim, et perveniet unus census qui est quattuor.

< 9 > Et similiter si dixerit: 'Est census cuius radix in quattuor radices eius multiplicata reddit triplum census et augmentum quinquaginta dragmarum.' Erit eius regula. Quoniam radix una in quattuor radices multiplicata facit quattuor census qui equantur triplo census illius radicis et quinquaginta dragmas. Ergo prohice tres census ex quattuor censibus. Et remanebit census qui erit equalis quinquaginta dragmis. Ipse enim est census. Cum ergo multiplicabis radicem quinquaginta in radices quattuor quinquaginta, proveniet triplum census et augmentum quinquaginta dragmarum.

< 10 > Quod si dixerit tibi: 'Est census cui addidi viginti dragmas, et fuit quod provenit equale duodecim radicibus census.' Erit eius regula. Quoniam dicis quod census et viginti equantur duodecim radicibus. Ergo media radices et multiplica eas in se, et minue ex eis viginti dragmas, et assume radicem eius quod remanet. Ipsam ergo ex

26 que] quem *P* et²] si *L* 28 viginti] 10 *F* quoniam cum *om. L* tu multiplicas] ut multiplices *L* 32 triginta²] 20 *F* 51 census² *om. F* 53 radicem] radices *L* 53-54 radices quattuor quinquaginta] quattuor eius radicum 200 *M* 54 *post* proveniet *add.* quam sunt *M* 57 *post* radicibus *add.* census *C*

medietate radicum que est sex minue. Quod igitur remanet est radix census, quod est duo. Et census est quattuor.

< 11 > Si vero dixerit: 'Multiplicavi tertiam census in quartam ipsius, et rediit census.' Erit eius regula. Quoniam cum multiplicas tertiam rei in quartam rei, provenit medietas sexte census que equatur rei. Ergo census est duodecim res. Et ipse est census.

< 12 > Quod si tibi dixerit: 'Est census cuius tertiam et dragmam multiplicavi in quartam ipsius et duas dragmas, et rediit census et augmentum tredecim dragmarum.' Erit eius consideratio ut multiplices tertiam rei in quartam rei et proveniet medietas sexte census, et dragmam in quar(P 116ra)tam rei et proveniet quarta rei, et duas dragmas in tertiam rei et provenient due tertie rei, et dragmam in duas dragmas et erunt due dragme. Erit ergo totum illud medietas sexte census et due dragme et undecim partes duodecim ex radice, que equantur radici et tredecim dragmis. Prohice ergo duas dragmas ex tredecim et remanebunt undecim. Et prohice undecim partes ex radice, et remanebit medietas sexte radicis et undecim dragme, qui equantur medietati sexte census. Ipsum ergo reintegra quod est, ut ipsum in duodecim multiplices et multiplices omne quod est cum eo in duodecim. Proveniet ergo quod census equatur centum et triginta duabus dragmis et radici. Oppone ergo per ea.

< 13 > Quod si dixerit: 'Est census cuius tertiam et quartam proieci, et insuper quattuor dragmas. Et multiplicavi quod remansit in se. Et quod provenit fuit equale censui et augmento duodecim dragmarum.' Huius regula erit ut accipias rem et auferas tertiam et quartam ex eo, et remanebunt quinque duodecime partes rei. Et minue ex eis quattuor dragmas, et remanebunt quinque duodecime partes rei exceptis quattuor dragmis. Eas igitur in se multiplica. Erunt ergo quinque partes in se multiplicate, viginti quinque partes centessime quadragesime quarte census. Postea multiplica quattuor dragmas exceptas in quinque partes duodecimas rei duabus vicibus. Et erunt quadraginta partes, quarum queque duodecim sunt res una. Et quattuor dragme diminute in quattuor fiunt sedecim dragme addite. Fiunt ergo quadraginta partes, tres radices et tertia radicis diminute. Proveniunt ergo tibi viginti quinque partes centesime quadragesime quarte census et sedecim dragme exceptis tribus radicibus et tertia, que equantur radici et duodecim dragmis. Per eas igitur oppone. Prohice igitur duodecim ex sedecim et remanent quattuor dragme. Et adde tres radices et tertiam radicis et provenient tibi quattuor radices et tertia radicis que equantur viginti quinque partibus centesimis quadragesimis quartis census et quattuor dragmis. Oportet igitur ut censum tuum reintegres. Ipsum ergo multiplica in quinque et decem et novem partes vigesimas quintas donec reintegretur. Et multiplica quattuor dragmas in quinque et decem et novem partes. Erunt ergo viginti tres dragme et pars una vigesima quinta. Et multiplica quattuor radices et tertiam in quinque et decem et novem partes vigesimas quintas. Erunt ergo viginti quattuor radices et viginti quattuor partes vigesimas quintas radicis.

67 in *om. L* 71 undecim²] 2 *F* ex radice *om. F* 78 duodecim] 13 *F* 79 partes *om. F* 79-80 et³ ... rei *om. F* 87 tertia] tertiam *L* 87-88 exceptis ... dragmis *om. FM* 93 *post* dragmas *add.* tres *L* 94-95 erunt ... quintas *om. M* 94 tres *om. L* 93-94 decem ... partes] 19/25 partes dragmarum *F*: et in radices et 3 in 5 et 19/25 *M*

Media ergo radices. Erunt ergo duodecim radices et duodecim partes vigesime quinte. Multiplica ergo eas in se, et erunt centum et quinquaginta quinque et quadringente et sexaginta novem partes sexcentesi(P 116rb)me et vigesime quinte. Minue ergo ex eis viginti tres et partem vigesimam quintam que est cum censu. Et remanebunt centum et triginta duo et quadraginte et quadringenta quattuor partes sexcentesime et vigesime quinte. Eius itaque accipe radicem que est undecim et tredecim partes vigesime et quinte. Ipsam ergo medietati radicum, que est duodecim et duodecim partes vigesime quinte, adde. Erit ergo illud viginti quattuor, qui est census quem queris.

< 14 > Si vero tibi dixerit: 'Est census quem in duas tertias multiplicavi et provenit quinque.' Erit eius consideratio ut multiplices rem aliquam in duas tertias rei et sint due tertie census equales quinque. Ipsam ergo reintegra per equalitatem medietatis ipsius, et adde super quinque ipsius medietatem. Et habebis censum equalem septem et semis. Radix ergo eius est res quam multiplicabis in duas tertias et proveniet quinque.

< 15 > Quod si dixerit tibi: 'Duo census sunt inter quos sunt due dragme. Quorum minorem per maiorem divisi, et evenit ex divisione medietas.' Erit eius regula ut multiplices rem et duas dragmas in id quod ex divisione provenit quod est medietas; et erit quod proveniet medietas rei et dragma que equantur rei. Prohice ergo medietatem cum medietate. Remanet dragma que equatur medietati rei. Duplica eas. Ergo habebis rem que equatur duabus dragmis, et ipsa est unus duorum censuum. Et alter census est quattuor.

< 16 > Si autem dixerit: 'Multiplicavi censum *in tres radices et provenit quintuplum census.' Quod est quasi dixisset: 'Multiplicavi censum* in radicem suam et fuit quod provenit equale censui et duabus tertiis. Ergo radix census est dragma et due tertie. Et census est due dragme et septem none.'

< 17 > Quod si dixerit tibi: 'Est census cuius proieci tertiam. Deinde multiplica residuum in tres radices census primi, et rediit census primus.' Erit eius regula. Quoniam cum tu multiplicas totum censum ante proiectionem sue tertie in tres radices eius, provenit census et semis, quoniam due tertie eius multiplicate in tres radices eius faciunt censum. Ergo ipse totus multiplicatus in tres radices eius est census et semis. Ipse ergo totus multiplicatus in radicem unam reddit census medietatem. Ergo radix census est medietas. Et census est quarta. Tertie ergo census due sunt sexta. Et tres radices census est dragma et semis. Quotienscumque igitur multiplicas sextam in dragmam et semis, provenit quarta que est census tuus.

< 18 > Sin autem dixerit: 'Est census cui abstuli quattuor radices. Deinde accepi tertiam residui, que fuit equalis quattuor radicibus. Census igitur est ducenta et quinquaginta sex.' Erit eius regula. Quia enim scis quod tertia eius quod remanet est equale quattuor radicibus eius, et sic illud quod remanet est equale duodecim radicibus. Ergo adde ei quattuor radices quas prius abstulisti, et erit sedecim radices. Ipse enim est radix census.

< 19 > (P 116va) Quod si dixerit: 'Est census de quo radicem suam proieci et addidi radici radicem eius quod remansit, et quod provenit fuit due dragme. Ergo hec radix census et radix eius quod remansit fuit equale duabus dragmis.' Prohice ergo ex duabus dragmis radicem census. Erunt itaque due dragme excepta radice in se multiplicate, quattuor dragme et census exceptis quattuor radicibus. Que equantur censui radice diminuta. Oppone ergo per eas. Est ergo census et quattuor dragme que equantur censui et tribus radicibus. Prohice itaque censum cum censu, et remanebunt tres radices equales quattuor dragmis. Ergo radix equatur dragme et tertie. Et census est dragma et septem none dragme unius.

< 20 > Et si dixerit: 'Est census ex quo proieci tres radices suas. Deinde residuum in se multiplicavi et provenit census.' Iam ergo scis quod illud quod remanet est etiam radix, et quod census est quattuor radices. Et ipse est sedecim dragme.

< 21 > Si quis autem tibi dixerit: 'Multiplicavi censum in duas tertias ipsius et provenit quinque.' Erit eius regula. Quoniam cum multiplicas ipsum in se, provenit septem et semis. Dic ergo quod ipse est radix septem et semis. Multiplica igitur duas tertias radicis septem et semis, quod est ut multiplices duas tertias in duas tertias. Provenient ergo quattuor none. Quattuor ergo none multiplicate in septem et semis sunt tres et tertia. Ergo radix trium et tertia est due tertie radicis septem et semis. Multiplica igitur tres et tertiam in septem et semis. Et provenient viginti quinque dragme, cuius radix est quinque.

99 sexcentesime et vigesime quinte] 500 *F* 103 duodecim[1] ... partes] 13 partes *F* 105-116 si ... quattuor *om. M* 115-116 et[2] ... quattuor *om. ms. Paris lat. 7377A* 118 *post* suam *add.* et multiplicavi censum in radicem suam *L* 119 ergo ... tertie *om. F* census ... tertie *om. L* 120 due *bis P* 122-124 quoniam ... semis *om. M* 123 multiplicas] multiplices *L* 124 provenit ... eius[3] *om. C*

140 quattuor[1]] tunc erunt F 141 diminuta *om. F* est] et *L* 142 tribus] duobus *F* 144 unius] unias *L* 148 multicplicavi *om. F* 150 dic ... semis *om. L* 152 none[1] *om. FM*

Glossary of L.

With few exceptions, the locations o· occur. If a word has a second use or meaning, its locatReferences are to chapters (A = Appendix) and line numb

accipere, *see* radicem accipere. :nber by another
addere (II.B.7), to add.
additio (II.B.50), addition. unit(s).
adiungere (III.50), act of adding one fignumber.
another. the doubling of
aggregare (II.B.8), to add.
aggregatio (V.1), sum.
algebra (*titulus operis* 2), neither this wor
the next is formally defined.
almuchabala (*titulus operis* 2), *see* algebı
angulus (III.54), angle.
appretiatum (VIII.5), a quantity to be length.
chased, known or unknown (V.ine segment, in-
according to its position in a proport
articulus (IV.9), two-digit number.
auferre (A 130), to take away, subtract. .eneric term for
augmentare (II.A.18), to increase. ing two terms
augmentatus (II.A.18), a coefficient g
than one. 'n length of a
augmentum (II.B.56), an increase. known number
cafficius (VIII.20), unit of liquid measur
cambitio (VIII.3), exchange. ation impossible
cambium (VIII.58), change.
causa (III.2), reason, explanation.
census (I.19 – definition), a square nuuadrilateral.
(*but see* p. 218 n. 16 above). e figure, a line
complere, *see* quadratum complere *and*
cem complere.
computatio (I.5), computation. . quadrilateral.
conductio (VIII.3), payment. ıt is greater than
coniungere (V.90), to join (particularly,
segments). (*see* next two
conventiones negociationis (VIII.2), con
cial methods. If the coefficient
demonstrare (III.80), to show or explaiı
not to prove. ılf the coefficient
diminuere (IV.12), to subtract.
diminutio (II.B.51), subtraction. ing.
diminutus (IV.12), a coefficient less than.
also missing or removed (III.14), *als*cent is less than
participle of *diminuere* (IV.12).
dividere (III.58), to cut a line segmenf number.

multiplicare (II.B.9), to multiply; *in se* means to square (I.18).
negociatio (VIII.1), *see* conventiones.
notus (V.10), known (in the sense of rational number).
numerus (I.6 – definition), the specified number of a quantity.
numerus simplex (I.20-21 – definition), constant.
opponere (VI.74), to subtract a positive term on one side of an equation from its larger like term on the other side; *also* in problems involving proportions, given a:b = c:d, then *a* opposes *d* and *b* opposes *c* (VIII passim).
pars (VI.8), part of a number.
pauciores (II.B.13), the coefficient of the square is less than one.
plures (II.B.13), the coefficient of the square is more than one.
ponderatio (VIII.59), weighing.
pretium (VIII.5), a price known *or* unknown (VIII.34), according to its position in a proportion.
pro(h)icere (VI.54), to subtract (used together with *opponere*).
proiectio (V.65), subtraction.
proportio (I.21), relationship or ratio *but not* proportion.
protrahere (V.60), to extend.
punctum (V.59), point.
quadrare (III.61), to create a square figure.
quadratura (III.28), square area; *also* a square.
quadraturam complere (III.16), to bring a polygon into the shape of a square by adding one or more squares to it.
quadratum complere (III.43), *see preceding phrase*.
qualiter (IV.2), in which way.
quantitas (IV.7), the number of.
questio (II.B.9), equation.
radix (I.17 – definition), the root of the square in the problem.
radicem accipere (II.B.11), to find the square root of a number.
radicem complere (A 21), to reduce or increase the coefficient of the root to one; *analogous to* reducere.
reducere (II.A.18), to bring the coefficient of a square term to unity by multiplying it by its multiplicative inverse.
regula (II.B.8), procedure.
reintegrare (II.B.31), *same as* reducere.
reintegratio (VI.57), *noun for the preceding verb*.
res (IV.31), an unknown quantity, often a first degree variable.
residuum (V.68), remainder.
restaurare (V.98), roots opposite in sign are added together; a negative term on one side of an equation is transferred to the other side (VI.18), which is the meaning of *restaurare* hereafter.
secare (III.86), to cut or divide a line segment.
sectio (VI.10), part of a number.
sensibilis (V.94), visual.
significatio (II.B.16), meaning.
superficies quadrata (III.3), a square.
superfluum (III.76), excess; *also* difference (VII.23).
surdus (V.10), surd (*in the sense of irrational number*).
totus (III.83), complete, entire.
triplicare (I.11), to triple a number.
triplum (V.14), thrice.
unitas (IV.8), one-digit number.
venditio (VIII.3), selling.

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